A FUZZY NEUTROSOPHIC SOFT MATRIX APPROACH IN

DECISION MAKING

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Abstract: The focus of this paper is to present a new technique for handling decision making problems in the light of Fuzzy Neutrosophic soft sets. We have proposed some new notions on matrix representation. Further we introduce a new score function in order to evaluate the degree of suitability of the choice of a certain alternative.

Keywords: Fuzzy Neutrosophic soft matrices, Cardinal set of Fuzzy Neutrosophic soft set and aggregate operator of Fuzzy Neutrosophic soft set.

MSC 2000: 03B99, 03E99

1. Introduction

Zadeh’s classical concept of fuzzy sets [17] is a strong mathematical tool to deal with information which is sometimes vague, sometimes inexact or imprecise and occasionally insufficient. Among several higher order fuzzy sets, intuitionistic fuzzy sets [IFS] introduced by Atanassov[2] have been found to be very useful and applicable. In the year 1999 Molodtsoy [12] presented soft set as a completely generic mathematical tool for modeling uncertainties. Since there is hardly any limitations in describing the objects, researchers simplify the decision making process by selecting the form of parameters they require and subsequently makes it more efficient in the absence of partial information. In [6,8] Maji,et.al initiated the concept of fuzzy soft sets and intuitionistic soft sets.

The concept of Neutrosophic set was initiated by [14,15] Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. Maji[9] extended soft sets to Neutrosophic soft set and applied in decision making problem. In this paper we introduce Fuzzy Neutrosophic matrix model with some new definitions. Further we have applied our approach in decision making problem.

2. PRELIMINARIES

Definition 2.1: [12]
Suppose $U$ is an universal set and $E$ is a set of parameters, Let $P(U)$ denotes the power set of $U$. A pair $(F, E)$ is called a soft set over $U$ where $F$ is a mapping given by $F: E \to P(U)$. Clearly, a soft set is a mapping from parameters to $P(U)$ and it is not a set, but a parameterized family of subsets of the universe.

**Definition 2.2:**[6]

Let $U$ be an initial universe set and $E$ be the set of parameters. Let $A \subseteq E$. A pair $(F, A)$ is called fuzzy soft set over $U$ where $F$ is a mapping given by $F: A \to I^U$ where $I^U$ denotes the collection of all fuzzy subsets of $U$.

**Definition 2.3:**[10]

Let $U = \{c_1, c_2, \ldots, c_m\}$ be the universal set and $E$ be the set of parameters given by $E = \{e_1, e_2, \ldots, e_n\}$. Let $A \subseteq E$. A pair $(F, A)$ be a fuzzy soft set in the fuzzy soft class$(U,E)$. Then fuzzy soft set $(F, A)$ in a matrix form as $A_{m \times n} = [a_{ij}]_{m \times n}$ or $A = [a_{ij}], i=1,2,\ldots,m, j = 1,2, \ldots, n$ where

$$
a_{ij} = \begin{cases} 
\mu_j(c_i) & \text{if } e_j \in A \\
0 & \text{if } e_j \not\in A
\end{cases}, \text{ and } \mu_j(c_i) \text{ represent the membership of } (c_i) \text{ in the fuzzy set } F(e_j).
$$

**Definition 2.4:**[8]

Let $U$ be an initial universe set and $E$ be the set of parameters. Let $A \subseteq E$. A pair $(F, A)$ is called intuitionistic fuzzy soft set over $U$ where $F$ is a mapping given by $F: A \to I^U$ where $I^U$ denotes the collection of all intuitionistic fuzzy subsets of $U$.

**Definition 2.5:**[11]

Let $U = \{c_1, c_2, \ldots, c_m\}$ be the universal set and $E$ be the set of parameters given by $E = \{e_1, e_2, \ldots, e_n\}$. Let $A \subseteq E$. A pair $(F, A)$ be a intuitionistic fuzzy soft set in the fuzzy soft class$(U,E)$. Then intuitionistic fuzzy soft set $(F, A)$ in a matrix form as $A_{m \times n} = [a_{ij}]_{m \times n}$ or $A = [a_{ij}], i=1,2,\ldots,m, j = 1,2, \ldots, n$ where

$$
da_{ij} = \begin{cases} 
\mu_j(c_i), \nu_j(c_i) & \text{if } e_j \in A \\
0, 1 & \text{if } e_j \not\in A
\end{cases}, \text{ where } \mu_j(c_i) \text{ represent the membership of } (c_i) \text{ and } \nu_j(c_i) \text{ represent the non-membership of } (c_i) \text{ in the intuitionistic fuzzy set } F(e_j).
$$

**Definition 2.6:**[1]

A Fuzzy Neutrosophic set $A$ on the universe of discourse $X$ is defined as

$$A = (x, T_A(x), I_A(x), F_A(x)), x \in X \text{ where } T, I, F: X \to [0, 1]$$

and
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0\leq T_A(x) + I_A(x) + F_A(x) \leq 3.

The set of all fuzzy neutrosophic set over the universe U will be denoted by FNS(U)

Definition 2.7:[1]

Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, \( A \subseteq E \). Let \( P(U) \) denotes the set of all fuzzy neutrosophic sets of U. The collection \( (F, A) \) is termed to be the fuzzy neutrosophic soft set over \( U \), where \( F \) is a mapping given by

\[ F: A \rightarrow P(U). \]

Throughout this paper Fuzzy Neutrosophic soft set is denoted by FNS set / FNSS. The set of all fuzzy Neutrosophic soft set over \( U \) will be denoted by \( (F,A)(U) \)

3. FUZZY NEUTROSOFTIC SOFT MATRIX THEORY

Definition 3.1:

Let \( U = \{ c_1, c_2, \ldots, c_m \} \) be the universal set and \( E \) be the set of parameters given by \( E = \{ e_1, e_2, \ldots, e_n \} \). Let \( A \subseteq E \). A pair \( (F, A) \) be a Fuzzy Neutrosophic soft set over \( U \). Then the subset of \( U \times E \) is defined by \( R_A = \{(u, e); e \in A, u \in f_A(e)\} \) which is called a relation form of \( (f_A, E) \). The membership function, indeterminacy membership function and non-membership function are written by \( T_{RA} : U \times E \rightarrow [0,1], I_{RA} : U \times E \rightarrow [0,1] \) and \( F_{RA} : U \times E \rightarrow [0,1] \) where \( T_{RA}(u,e) \in [0,1] \), \( I_{RA}(u,e) \in [0,1] \) and \( F_{RA}(u,e) \in [0,1] \) are the membership value, indeterminacy value and non-membership value respectively of \( u \in U \) for each \( e \in E \).

If \( [T_{ij}, I_{ij}, F_{ij}] = (T_{ij}(u_1, e_j), I_{ij}(u_1, e_j), F_{ij}(u_1, e_j)) \) we can define a matrix

\[
\begin{bmatrix}
T_{11}' & I_{11}' & F_{11}' \\
T_{12}' & I_{12}' & F_{12}' \\
\vdots & \vdots & \vdots \\
T_{mn}' & I_{mn}' & F_{mn}'
\end{bmatrix}
\]

which is called an m x n Fuzzy Neutrosophic Soft Matrix of the FNSS \((f_A, E)\) over \( U \).

Definition 3.2:

Let \( U = \{ c_1, c_2, \ldots, c_m \} \) be the universal set and \( E \) be the set of parameters given by \( E = \{ e_1, e_2, \ldots, e_n \} \). Let \( A \subseteq E \). A pair \( (F, A) \) be a fuzzy neutrosophic soft set. Then fuzzy neutrosophic soft set \( (F, A) \) in a matrix form as \( A_{m \times n} = [a_{ij}]_{m \times n} \) or \( A = [a_{ij}], i=1,2,\ldots,m, j=1,2,\ldots,n \) where
\[
\mathbf{a}_{ij} = \begin{cases} 
T_j(c_i), I_j(c_i), F_j(c_i) & \text{if } e_j \in A \\
(0, 0, 1) & \text{if } e_j \notin A 
\end{cases}
\]

where \(T_j(c_i)\) represent the membership of \((c_i, 1)\), \(I_j(c_i)\) represent the indeterminacy of \((c_i, j)\) and \(F_j(c_i)\) represent the non-membership of \((c_i, j)\) in the fuzzy neutrosophic set \(F(e_j)\).

**Note:** Fuzzy Neutrosophic soft matrix is denoted by FNSM

**Example 3.3:**

Suppose that \(U = \{s_1, s_2, s_3, s_4\}\) is a set of cars and \(E = \{e_1, e_2, e_3\}\) is a set of parameters, which stand for mileage, engine and maintenance respectively. Consider the mapping from parameters set \(A \subseteq E\) to the set of all fuzzy neutrosophic subsets of power set \(U\). Then the set \((F, A)\) describes the characteristic of the cars with respect to the given parameters.

Consider \(A = \{e_1, e_2\}\) then FNSS is

\[
(F, A) = \begin{bmatrix} 
{F(e_1)} = \{(s_1, 0.8, 0.5, 0.4), (s_2, 0.3, 0.5, 0.6), (s_3, 0.8, 0.7, 0.3), (s_4, 0.9, 0.1, 0.0)\} \\
{F(e_2)} = \{(s_1, 0.8, 0.5, 0.4), (s_2, 0.5, 0.6, 0.3), (s_3, 0.4, 0.5, 0.5), (s_4, 0.3, 0.4, 0.6)\}
\end{bmatrix}
\]

We represent the above as Fuzzy neutrosophic soft matrix form.

\[
(F, A) = \begin{bmatrix} 
\mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\
\mathbf{s}_1 & (0.8, 0.5, 0.4) & (0.8, 0.5, 0.4) & (0, 0, 1) \\
\mathbf{s}_2 & (0.3, 0.5, 0.6) & (0.5, 0.6, 0.3) & (0, 0, 1) \\
\mathbf{s}_3 & (0.8, 0.7, 0.3) & (0.4, 0.5, 0.5) & (0, 0, 1) \\
\mathbf{s}_4 & (0.9, 0.1, 0.0) & (0.3, 0.4, 0.6) & (0, 0, 1)
\end{bmatrix}
\]

**Definition 3.4:**

Let \(\widetilde{\mathbf{A}} = \{\mathbf{A}, \mathbf{I}, \mathbf{F}\} \subseteq \text{FNSM}_{m \times n}\) then \(\widetilde{\mathbf{A}}\) is called

\begin{itemize}
  \item[a)] A zero or null FNSM denoted by \(\mathbf{\varnothing} = [0, 0, 1]\) if \(T_{ij} = 0\), \(I_{ij} = 0\) and \(F_{ij} = 1\), \(\forall i \text{ and } j\). It is denoted by \(\varnothing\).
  \item[b)] A universal FNSM denoted by \(\mathbf{I} = [1, 1, 0]\) if \(T_{ij} = 1\), \(I_{ij} = 1\) and \(F_{ij} = 0\), \(\forall i \text{ and } j\). It is denoted by \(\mathbf{U}\).
\end{itemize}

**Definition 3.5:**

Let \(\tilde{\mathbf{A}} = \{\tilde{\mathbf{A}}, \tilde{\mathbf{I}}, \tilde{\mathbf{F}}\} \subseteq \text{FNSM}_{m \times n}\) and \(\tilde{\mathbf{B}} = \{\tilde{\mathbf{B}}, \tilde{\mathbf{I}}, \tilde{\mathbf{F}}\} \subseteq \text{FNSM}_{m \times n}\). Then

\begin{itemize}
  \item[a)] \(\tilde{\mathbf{A}}\) is Fuzzy neutrosophic soft sub matrix of \(\tilde{\mathbf{B}}\) denoted by \(\tilde{\mathbf{A}} \subseteq \tilde{\mathbf{B}}\) if \(T_{ij} \leq T_{ij}\), \(I_{ij} \leq I_{ij}\) and \(F_{ij} \geq F_{ij}\), \(\forall i \text{ and } j\).
\end{itemize}
Let \((F, A)\) be Fuzzy Neutrosophic soft set over \(U\). Then the cardinal set of \((F, A)\) denoted by \((F, A)\) is uniquely characterized by a matrix, which is the scalar cardinalities of the FNSS respectively. The set of all cardinal sets of FNSS over \(U\) will be denoted by \((F, A)\) for FNNS over \(U\). Let \((F, A)\) be FNSS over \(U\). Let \((F, A)\) be the cardinal set over \(U\). Suppose \(E = \{e_1, e_2, ... , e_n\}\) and \(A\subseteq E\) then \((F, A)\) can be represented by the following table.

<table>
<thead>
<tr>
<th>(E)</th>
<th>(e_1)</th>
<th>......</th>
<th>(e_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>({T_{e_1}(x), I_{e_1}(x), F_{e_1}(x)})</td>
<td>({T_{e_2}(x), I_{e_2}(x), F_{e_2}(x)})</td>
<td>......</td>
<td>({T_{e_n}(x), I_{e_n}(x), F_{e_n}(x)})</td>
</tr>
</tbody>
</table>

If \(\{T_{e_j}(x, y), F_{e_j}(x, y)\}_{j=1}^{n}\) = \(\{T_{e_j}(x, y), F_{e_j}(x, y)\}_{j=1}^{n}\) for \(j = 1, 2, ... , n\) then the cardinal set \((F, A)\) is uniquely characterized by a matrix,

\[\{T_{e_j}(y, y), F_{e_j}(y, y)\}_{j=1}^{n} = [(T_{e_1}, I_{e_1}, F_{e_1}), (T_{e_2}, I_{e_2}, F_{e_2}) , ... , (T_{e_n}, I_{e_n}, F_{e_n})]\]

which is called the cardinal matrix of the cardinal set \((F, A)\) over \(E\).

**Definition 3.8:**

Let \((F, A)\) be FNNS over \(U\). Let \((F, A)\) be the cardinal set over \(U\). Then the aggregation operator, denoted by \((F, A)\) defined by \((F, A)\) is a function from \((F, A)\) to \(FNS(U)\) such that \((F, A)\) \(=(F, A)\) where 

\[\{T_{e_j}(y, y), F_{e_j}(y, y)\}_{j=1}^{n} = [(T_{e_1}, I_{e_1}, F_{e_1}), (T_{e_2}, I_{e_2}, F_{e_2}) , ... , (T_{e_n}, I_{e_n}, F_{e_n})]\]
(F,A) = \left\{ (T, I_i, F_i) : u \in U \right\} is an fuzzy Neutrosophic set over U. Here (F,A) is called the aggregate fuzzy neutrosophic set of the FNSS (F,A). The membership function $T_{\xi^+}$, indeterministic function $I_{\xi^+}$ and non-membership function $F_{\xi^+}$ are defined as follows.

- $T_{\xi^+} : U \rightarrow [0,1]$ such that $T_{\xi^+}(u) = \frac{1}{|E|} \sum_{e \in E} T_{(F,A),e}(u) \cdot T_{\xi^+}(u)$
- $I_{\xi^+} : U \rightarrow [0,1]$ such that $I_{\xi^+}(u) = \frac{1}{|E|} \sum_{e \in E} I_{(F,A),e}(u) \cdot I_{\xi^+}(u)$
- $F_{\xi^+} : U \rightarrow [0,1]$ such that $F_{\xi^+}(u) = \frac{1}{|E|} \sum_{e \in E} F_{(F,A),e}(u) \cdot F_{\xi^+}(u)$

where $|E|$ is the cardinality of E.

**Definition 3.9:**

Let (F,A) be FNSS and (F,A) be its aggregate Neutrosophic set and $U = \{u_1, u_2, \ldots, u_n\}$, then (F,A) can be presented by the following table.

<table>
<thead>
<tr>
<th>(F,A)</th>
<th>(T_{\xi^+}, I_{\xi^+}, F_{\xi^+})</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>({T_{\xi^+}(u_1), I_{\xi^+}(u_1), F_{\xi^+}(u_1)})</td>
</tr>
<tr>
<td>$u_2$</td>
<td>({T_{\xi^+}(u_2), I_{\xi^+}(u_2), F_{\xi^+}(u_2)})</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>$u_m$</td>
<td>({T_{\xi^+}(u_m), I_{\xi^+}(u_m), F_{\xi^+}(u_m)})</td>
</tr>
</tbody>
</table>

If \(\{T_{i1}, I_{i1}, F_{i1}\} = \{T_{\xi^+}(u_i), I_{\xi^+}(u_i), F_{\xi^+}(u_i)\}\) for $i = 1, 2, \ldots, m$ then (F,A) is uniquely characterized by a matrix,

\[
[T_{i1}, I_{i1}, F_{i1}]_{m\times1} =
\begin{bmatrix}
(T_{11}, I_{11}, F_{11}) \\
(T_{21}, I_{21}, F_{21}) \\
\vdots \\
(T_{m1}, I_{m1}, F_{m1})
\end{bmatrix}
\]
which is called the aggregate matrix \((F,A)^+\) over \(U\).

**Definition 3.10:**

If \(M(F,A)\), \(M(F,A)_C\) and \(M(F,A)^+\) are representation matrices of \((F,A)\), \((F,A)_C\) and \((F,A)^+\) respectively where \((F,A)\) is FNSS over \(U\) and \(A \subseteq E\) only if

\[
M(F,A)^+ = \frac{1}{|E|} M(F,A) \cdot M(F,A)_C^T,
\]

where \(M(F,A)_C^T\) is the transposition of \(M(F,A)_C\).

**Definition 3.11:**

The score value of \(S_i\) for \(u_i \in U\) is defined as

\[
S_i = \sum_{u_i \in U} (T_i - I_i,F_i).
\]

**Algorithm:**

1. Construct an FNSS \((F,A)\) over \(U\)
2. Compute the cardinal set \((F,A)_C\) of \((F,A)\).
3. Compute aggregate FNS \((F,A)^+\) of \((F,A)\).
4. Compute the score value of \(S_i\) for each \(u_i \in U\) from the set \((F,A)^+\).
5. Find \(S_j = \max(S_i)\), then we conclude that \(c_i\) is suitable.

**Application in decision making:**

Suppose an investor wants to invest a part of his money in a company in order to get high profits. If all the companies process are almost same, it is rather very difficult for the investor to select a company right way.

Let us consider the parameter be \(E = \{\text{constant growth rate}(e_1), \text{negative growth rate}(e_2), \text{low growth rate}(e_3), \text{medium growth rate}(e_4), \text{high growth rate}(e_5)\}\).

Suppose the investor wants to select a company for the view points of parameter \(e_1, e_4\) and \(e_5\) i.e., \(A = \{e_1, e_4, e_5\}\). There are five companies who form the set of universe \(U = \{c_1, c_2, c_3, c_4, c_5\}\) where \(c_1\) is a medicine company, \(c_2\) is a software company, \(c_3\) is a car manufacturing company, \(c_4\) is a food processing company, \(c_5\) is an online shopping company.

**Step 1**

Then the investor constructs an FNSS \((F,A)\) to select a company which can provide more profit.

\[
(F,A) = \{(F(e_1)) = \{(c_1,0.5,0.6,0.3), (c_2,0.6,0.5,0.2), (c_3,0.4,0.6,0.4), (c_4,0.7,0.4,0.2), (c_5,0.8,0.4,0.2)\} \}
\]

\[
(F(e_4)) = \{(c_1,0.7,0.5,0.1), (c_2,0.3,0.6,0.5), (c_3,0.5,0.4,0.4), (c_4,0.3,0.7,0.6), (c_5,0.6,0.5,0.4)\} \}.
\]
F(ε5) = {(ε1,0.5,0.5,0.5), (ε2,0.4,0.7,0.5), (ε3,0.6,0.5,0.3),
(ε4,0.7,0.4,0.1), (ε5,0.4,0.4,0.5)}

The tabular representation of the FNSS is

<table>
<thead>
<tr>
<th></th>
<th>ε1</th>
<th>ε4</th>
<th>ε5</th>
</tr>
</thead>
<tbody>
<tr>
<td>c1</td>
<td>(0.5,0.6,0.3)</td>
<td>(0.7,0.5,0.1)</td>
<td>(0.5,0.5,0.5)</td>
</tr>
<tr>
<td>c2</td>
<td>(0.6,0.5,0.2)</td>
<td>(0.3,0.6,0.5)</td>
<td>(0.4,0.7,0.5)</td>
</tr>
<tr>
<td>c3</td>
<td>(0.4,0.6,0.4)</td>
<td>(0.5,0.4,0.4)</td>
<td>(0.6,0.5,0.3)</td>
</tr>
<tr>
<td>c4</td>
<td>(0.7,0.4,0.2)</td>
<td>(0.3,0.7,0.6)</td>
<td>(0.7,0.4,0.1)</td>
</tr>
<tr>
<td>c5</td>
<td>(0.8,0.4,0.2)</td>
<td>(0.6,0.5,0.4)</td>
<td>(0.4,0.4,0.5)</td>
</tr>
</tbody>
</table>

Step 2:
The cardinality of FNSS (F,A) is
(F,A)C = {(0.6,0.5,0.26)/ε1 , (0.48,0.54,0.4)/ε4 , (0.52,0.5,0.38)/ε5}

Step 3:

\[
M(F,A)^+ = \frac{1}{|E|} M(F,A) \cdot M(F,A)^T_C^{-}
\]

\[
M(F,A)^+ = \begin{bmatrix}
(0.1792, 0.1640, 0.4616) \\
(0.1424, 0.1848, 0.4884)
\end{bmatrix}
\]

\[
(F,A)^+ = \{ (0.1792, 0.1640, 0.4616)/c_1 , (0.1424, 0.1848, 0.4884)/c_2,
(0.1584, 0.1532, 0.4758)/c_3, (0.1856, 0.1556, 0.4660)/c_4,
(0.1952, 0.1340, 0.4804) /c_5 \}
\]

Step 4:
Compute the score function s_i for each c_i.

\[
S_i = \sum_{u \in F_j} (T_{i,j} - I_{j,F_i})
\]

S_1 = 0.5203, S_2 = 0.3363, S_3 = 0.4163, S_4 = 0.5523, S_5 = 0.6003.

Here S_3 is maximum. Therefore c_3 is the best alternative for the investor.

4. CONCLUSION:
Soft set theory is a general method for solving problems of uncertainty. In this paper, we define Fuzzy Neutrosophic soft matrices and some new definitions which are more functional to make theoretical studies in the Fuzzy Neutrosophic soft set theory. Further we have applied the notion of FNSM in a decision making problem.

References:
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