FUZZY NEUTROSOPOHIC ROUGH SETS
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Abstract: In this paper, we introduce Fuzzy Neutrosophic rough set and study some basic operations. Further we develop a systematic study on Fuzzy Neutrosophic rough set and obtain various properties induced by them. Some equivalent characterization and inter-relations among them are discussed.

Keywords: Rough set, Neutrosophic set, Fuzzy Neutrosophic sets, Fuzzy Neutrosophic rough sets

INTRODUCTION
Rough set theory is a [7], is an extension of set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. Moreover, it is a mathematical tool for machine learning, information sciences and expert systems and successfully applied in data analysis and data mining. There are two basic elements in rough set theory, crisp set and equivalence relation, which constitute the mathematical basis of RSs. The basic idea of rough set is based upon the approximation of sets by a pair of sets known as the lower approximation and the upper approximation of a set. In classical rough set theory partition or equivalence relation is the basic concept.

Now fuzzy sets are combined with rough sets in a fruitful way and defined by rough fuzzy sets and fuzzy rough sets [4, 5][10-12]. Also fuzzy rough sets, generalize fuzzy rough, intuitionistic fuzzy rough sets, rough intuitionistic fuzzy sets, rough vague sets are introduced. The theory of rough sets is based upon the classification mechanism, from which the classification can be viewed as an equivalence relation and knowledge blocks induced by it be a partition on universe.

One of the interesting generalizations of the theory of fuzzy sets and intuitionistic fuzzy sets is the theory of neutrosophic sets introduced by F. Smarandache. Neutrosophic sets described by three functions: Truth function indeterminacy function and false function that are independently related. The theories of neutrosophic set have achieved great success in various areas such as medical diagnosis, database, topology, image processing, and decision making problem. While the neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data, the theory of rough sets is a powerful mathematical tool to deal with incompleteness.

Neutrosophic sets and rough sets are two different topics, none conflicts the other. Recently many researchers applied the notion of neutrosophic sets to relations, group theory, ring theory, Soft set theory and so on. In this paper we combine the mathematical tools fuzzy sets, rough sets and neutrosophic sets and introduce a new class of set called fuzzy neutrosophic rough sets. Here we give rough approximation of a fuzzy neutrosophic set and introduce fuzzy neutrosophic rough sets.
PRELIMINARIES:

Definition 2.1[1] A Neutrosophic set A on the universe of discourse X is defined as \( A= (x, T_A(x), I_A(x), F_A(x)) \), \( x \in X \). Where \( T, I, F: X \to [0,1]^* \) and \(-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+ \).

Definition 2.3: [1] A Fuzzy Neutrosophic set A on the universe of discourse X is defined as \( A= (x, T_A(x), I_A(x), F_A(x), x \in X \) where \( T, I, F: X \to [0, 1] \) and \( 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \).

Definition 2.2: [1] A neutrosophic set A is contained in another neutrosophic set B. (i.e.,) \( A \subseteq B \iff T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x) \forall x \in X. \)

Definition 2.4: [1] The complement of a neutrosophic set \((F,A)\) denoted by \((F,A)^c\) and is defined as \((F,A)^c = (F, |A|)\) where \( T_{F^c} (x) = F_F (x), I_{F^c} (x) = 1 - I_F (x), F_{F^c} (x) = T_F (x). \)

Definition 2.5: [1] Let A and B be two neutrosophic sets over the common universe U. A is said to be neutrosophic subset of B if \( A \subseteq B \) and \( T_A(x) \leq T_B(x), I_A(x) \leq I_B(x), F_A(x) \geq F_B(x) \forall x \in A, x \in U. \)

Definition 2.6: [1] Two neutrosophic sets \((F,A)\) and \((G,B)\) over the common universe U are said to be equal if \((F,A) \subseteq (G,B)\) and \((G,B) \subseteq (F,A)\). We denote it by \((F,A) = (G,B)\).

Definition 2.7: [1] Let X be a non empty set, and \( A = \langle x, T_A(x), I_A(x), F_A(x) \rangle \), \( B = \langle x, T_B(x), I_B(x), F_B(x) \rangle \) are fuzzy neutrosophic sets. Then

\[
A \cup B = \langle x, \max (T_A(x), T_B(x)), \max (I_A(x), I_B(x)), \min (F_A(x), F_B(x)) \rangle \\
A \cap B = \langle x, \min (T_A(x), T_B(x)), \min (I_A(x), I_B(x)), \max (F_A(x), F_B(x)) \rangle \\
\]

Example 2.8 Consider the following example:

Let A be set of four grocery stores. Let \( U = \{B_1, B_2, B_3, B_4\}. \)

Then fuzzy neutrosophic set representing their character is given by

\( F= \{(B_1, 0.8, 0.9, 0.1), (B_2, 0.7, 0.24, 0.05), (B_3, 0.85, 0.25, 0.6), (B_4, 0.7, 0.3, 0.2)\} \)

Where first numerical value in each bracket represents the truth value and the second and third represents indeterminate and false values respectively.
are called the lower and upper approximations respectively of X and the pair S=(U, R) is called the approximation space. The equivalence relation R is called indiscernibility relation.
The pair A(X) = (A₁(X), A₂(X)) is called the rough set of X in S. Here [x]ᵣ denotes the equivalence class of R containing x.

**Definition 2.3:** [2] Let A= (A₁, A₂) and B= (B₁, B₂) be two rough sets in approximation space S= (U, R). Then A∩B = (A₁∩B₁, A₂∩B₂)
A⊆B if A∩B=A.
A∩B = \{U-A₂, U-A₁\}

### 3 Fuzzy Neutrosophic Rough Sets

**Definition 3.1:** Let U be a non-null set and R be an equivalence relation on U. Let A be a fuzzy neutrosophic set in U with the truth value \( T_A(x) \), indeterminate value \( I_A(x) \) and false value \( F_A(x) \) and A = (x, \( T_A(x), I_A(x), F_A(x) \)), \( x \in X \) where \( T, I, F: X \to [0, 1] \) and \( 0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \).

The lower and the upper approximations of F in the approximation (U, R) denoted by \( \bar{R}(A) \) and \( \bar{\bar{R}}(A) \) are respectively defined as follows:
\[
\bar{R}(A) = \{ <x, T_{\bar{R}(A)}(y), I_{\bar{R}(A)}(y), F_{\bar{R}(A)}(y)> | y \in [x]_R, x \in U \}
\]
\[
\bar{\bar{R}}(A) = \{ <x, T_{\bar{\bar{R}}(A)}(y), I_{\bar{\bar{R}}(A)}(y), F_{\bar{\bar{R}}(A)}(y)> | y \in [x]_R, x \in U \},
\]
where
\[
T_{\bar{R}(A)} = \bigvee_{y \in [x]_R} T_A(y), I_{\bar{R}(A)} = \bigvee_{y \in [x]_R} I_A(y), F_{\bar{R}(A)} = \bigvee_{y \in [x]_R} F_A(y)
\]
\[
T_{\bar{\bar{R}}(A)} = \bigvee_{y \in [x]_R} T_A(y), I_{\bar{\bar{R}}(A)} = \bigvee_{y \in [x]_R} I_A(y), F_{\bar{\bar{R}}(A)} = \bigvee_{y \in [x]_R} F_A(y)
\]
So 0 \leq T_{\bar{R}(A)} + I_{\bar{R}(A)} + F_{\bar{R}(A)} \leq 3 and 0 \leq T_{\bar{\bar{R}}(A)} + I_{\bar{\bar{R}}(A)} + F_{\bar{\bar{R}}(A)} \leq 3 and \( T_{\bar{R}(A)}, I_{\bar{R}(A)}, F_{\bar{R}(A)}, T_{\bar{\bar{R}}(A)}, I_{\bar{\bar{R}}(A)}, F_{\bar{\bar{R}}(A)} : A \to [0, 1] \)

Where “\( V \) and “\( A \)“ mean “max” and “min “ operators respectively, , and are the truth, indeterminacy and false values of y with respect to A.

Thus, \( \bar{R}(A) \) and \( \bar{\bar{R}}(A) \) are two fuzzy neutrosophic sets in U.

\( \bar{R}(A), \bar{\bar{R}}(A): A \to A \) are, respectively, referred to as the fuzzy neutrosophic upper and lower rough approximation operators, and the pair \( \bar{R}(A), \bar{\bar{R}}(A) \) is called the fuzzy neutrosophic rough set in (U, R).

**Remark 3.2:** \( \bar{R}(A) \) and \( \bar{\bar{R}}(A) \) have constant membership on the equivalence classes of U.

ie, \( \bar{R}(A) = \bar{\bar{R}}(A) = T_{\bar{R}(A)} = T_{\bar{\bar{R}}(A)} \cdot I_{\bar{R}(A)} = I_{\bar{\bar{R}}(A)} \cdot F_{\bar{R}(A)} = F_{\bar{\bar{R}}(A)} \)

**Example 3.3:** Let U= \{S₁, S₂, S₃, S₄, S₅\} be the universe of discourse.

Let R be an equivalence relation, where its partition of U is given by
\[
U/R = \{ \{S₁, S₂\}, \{S₃\}, \{S₄, S₅\} \}
\]
A = \{| S₁,(0.3,0.4,0.5) | S₂,(0.2,0.4,0.3) | S₃,(0.5,0.6,0.7)\} be a neutrosophic set of U.
The lower and upper approximations are obtained as
\[ R(A) = \{ [S_1(0.3,0.4,0.3)] [S_2(0.3,0.4,0.3)] [S_3(0.5,0.6,0.7)] \} \]

Another fuzzy neutrosophic set can be defined as
\[ A = \{ [S_1(0.2,0.3,0.4)] [S_2(0.3,0.5,0.4)] [S_3(0.4,0.6,0.2)] \} \]
The lower and upper approximations are obtained as
\[ R(A) = \{ [S_1(0.3,0.5,0.4)] [S_2(0.4,0.6,0.2)] [S_3(0.4,0.6,0.2)] \} \]
\[ \overline{R}(A) = \{ [S_1(0.2,0.3,0.4)] [S_2(0.2,0.3,0.4)] \} \]

**Definition 3.4:** If \( A = (\overline{R}(A), R(A)) \) is a fuzzy neutrosophic rough set in \((U,R)\), then the rough complement of \( A \) is also a neutrosophic set denoted by

\[ \neg A = (\overline{R}(A)'', R(A)'') \], where \( \overline{R}(A)'', R(A)'' \) are the complements of neutrosophic sets \( \overline{R}(A), R(A) \) respectively.

\[ \overline{R}(A)'' = \{ <x,F_{\overline{R}(A)},1-I_{\overline{R}(A)},T_{\overline{R}(A)},\neg x> | x \in U \} \] and
\[ R(A)'' = \{ <x,F_{R(A)},1-I_{R(A)},T_{R(A)},\neg x> | x \in U \} \]

**Definition 3.5:** If \( A_1 \) and \( A_2 \) are two fuzzy rough neutrosophic set of the neutrosophic sets \( F_1 \) and \( F_2 \) respectively in \( (U,R) \) then we define the following:

1. \( A_1 = A_2 \) iff \( \overline{R}(A_1) = \overline{R}(A_2) \) and \( R(A_1) = R(A_2) \)
2. \( A_1 \subseteq A_2 \) iff \( \overline{R}(A_1) \subseteq \overline{R}(A_2) \) and \( R(A_1) \subseteq R(A_2) \)
3. \( A_1 \cup A_2 \) iff \( \overline{R}(A_1) \cup \overline{R}(A_2) \) and \( R(A_1) \cup R(A_2) \)
4. \( A_1 \cap A_2 \) iff \( \overline{R}(A_1) \cap \overline{R}(A_2) \) and \( R(A_1) \cap R(A_2) \)
5. \( A_1 + A_2 \) iff \( \overline{R}(A_1) + \overline{R}(A_2) \cdot \overline{R}(A_1) + \overline{R}(A_2) \)
6. \( A_1 \cdot A_2 \) iff \( \overline{R}(A_1) \cdot \overline{R}(A_2) \cdot \overline{R}(A_1) \cdot \overline{R}(A_2) \)

**Proposition 3.6:**

\( A_1, A_2 \) and \( A_3 \) are fuzzy rough neutrosophic set of the neutrosophic sets \( F_1 \) and \( F_2 \) respectively, then

1. \( \neg A_1 (\neg A_1) = A_1 \)
2. \( A_1 \cup A_2 = A_2 \cup A_1 \)
3. \( A_1 \cap A_2 = A_2 \cap A_1 \)
4. \( (A_1 \cup A_2) \cup A_3 = (A_1 \cup A_2) \cup A_3 \)
5. \( (A_1 \cap A_2) \cap A_3 = (A_1 \cap A_2) \cap A_3 \)
6. \( (A_1 \cup A_2) \cap A_3 = (A_1 \cup A_2) \cap (A_1 \cup A_3) \) and
7. \( (A_1 \cap A_2) \cup A_3 = (A_1 \cap A_2) \cup (A_1 \cap A_3) \)

De Morgan’s Laws are satisfied for neutrosophic sets

**Proposition 3.7:**

If \( A_1 \) and \( A_2 \) are fuzzy rough neutrosophic set then

1. \( \neg (A_1 \cup A_2) = (\neg A_1) \cap (\neg A_2) \)
2. \( \sim(A_1 \cap A_2) = (\sim A_1) \cup (\sim A_2) \)

**Proof:**

\[
\sim(A_1 \cup A_2) = \sim((R(A_1) \cup R(A_2), \overline{R}(A_1) \cup \overline{R}(A_2))
\]

\[
= \sim(\overline{R}(A_1) \cup \overline{R}(A_2))
\]

\[
= [\{\overline{0}(\square_1), \overline{0}(\square_2)\}^c, \{\overline{0}(\square_1), \overline{0}(\square_2)\}^c]
\]

\[
= [\{\overline{0}(\square_1), \overline{0}(\square_2)\} \cap \overline{0}(\square_2)], \{\overline{0}(\square_1), \overline{0}(\square_2)\}^c]
\]

\[
= (\sim \square_1) \cap (\sim \square_2)
\]

**Proposition 3.8:**

If \( A_1 \) and \( A_2 \) are two neutrosophic sets in \( U \) such that \( A_1 \subseteq A_2 \), then its fuzzy neutrosophic rough sets \( FA_1 \) and \( FA_2 \) is such that \( FA_1 \subseteq FA_2 \)

i. \( F(A_1 \cup A_2) = F(A_1) \cup F(A_2) \)

ii. \( F(A_1 \cap A_2) = F(A_1) \cap F(A_2) \)

**Proof:**

\[
\square \subseteq \overline{\square}(\square_1, \square_2, \square_3) \subseteq \inf \{\square_1(\square)/ \square \in [0,1], \square_2(\square)/ \square \in [0,1]\}
\]

\[
= \inf (\max \{\square_1(\square), \square_2(\square)/ \square \in [0,1]\})
\]

\[
\geq \max \{\inf(\square_1(\square)/ \square \in [0,1]), \inf(\square_2(\square)/ \square \in [0,1]\})
\]

\[
= \max \{\square_1(\square), \square_2(\square)/ \square \in [0,1]\}
\]

\[
= \{\square_1(\square), \square_2(\square)/ \square \in [0,1]\}
\]

Thus

\[
\square F(A_1 \cup A_2) \subseteq \square F(A_1) \cup \square F(A_2)
\]

we also have

\[
\square F(A_1 \cup A_2) = \square F(A_1) \cup \square F(A_2)
\]

Hence,

\[
F(A_1 \cup A_2) = FA_1 \cup FA_2
\]

Hence the proof (ii) is similar to (i).

**Proposition 3.7:**

Let \( A \) be a fuzzy neutrosophic rough set of a discourse \( U \), then

i) \( \square (A) = \sim \square (\sim A) \)

ii) \( \square (A)^N = \sim \square (\sim A) \)

iii) \( \square (A) = \square (A) \)

**Proof:**

i) \( A = \{x, T_A(x), I_A(x), F_A(x)/x \in X\} \)

\( \sim A = \{x, F_A(x), 1 - I_A(x), T_A(x)/x \in X\} \)
\[ \neg \overline{\neg (A)} = \{ <x, (\neg \overline{x}(\neg A)) : y \in [x]_\mathbb{R}, x \in U \} \]
\[ \neg \overline{\neg (\neg A)} = \{ <x, (\neg \overline{x}(\neg A)) : y \in [x]_\mathbb{R}, x \in U \} \]
\[ \overline{\neg (\neg A)} = \bigwedge_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \]
\[ \bigwedge_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) = V_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \]
Hence
\[ \overline{\neg (A)} = \neg \overline{\neg (\neg A)} \]

The proof (ii) is similar to (i)

For any \( y \in \overline{\neg (A)} \), we can have
\[ \overline{\neg (\neg A)} = \bigwedge_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \leq V_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \]
\[ \overline{\neg (\neg A)} = \bigwedge_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \leq V_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \]
\[ \overline{\neg (\neg A)} = V_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \geq \bigwedge_{x \in [x]_\mathbb{R}} \overline{\neg (\neg A)}(x) \]

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