PATH RELATED MEAN CORDIAL GRAPHS

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Abstract: Let \(G = (V, E)\) be a simple graph. \(G\) is said to be a mean cordial graph if \(f : V(G) \rightarrow \{0, 1, 2\}\) such that for each edge \(uv\) the induced map \(f^*\) defined by \(f^*(uv) = \left\lfloor \frac{f(u) + f(v)}{2} \right\rfloor\) where \(\lfloor x \rfloor\) denote the least integer which is \(\leq x\) and \(|e_0(0) - e_1(1)| \leq 1\) where \(e_0(0)\) is no.of edges with zero label. \(e_1(1)\) is no.of edges with one label.

The graph that admits a mean cordial labeling is called a mean cordial graph(MCG). In this paper, we proved that \(P_n \circ K_1\), \((P_n + K_1)\), \(P_n \times P_n\), \((P_n : C_3)\), \((P_n : S_1)\), \(P_n \times P_2\), \(P_n + 2K_1\), \(Z -(P_n)\) are mean cordial graphs.

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1. INTRODUCTION:

A graph \(G\) is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of \(G\) which is called edges. Each \(e = \{uv\}\) of vertices in \(E\) is called an edge or a line of \(G\). For graph theoretical Terminology we follow

2. PRELIMINARIES:

We define the concept of mean cordial labeling as follows. Let \(G = (V, E)\) be a simple graph. \(G\) is said to be a mean cordial graph if \(f : V(G) \rightarrow \{0, 1, 2\}\)

Such that for each edge \(uv\) the induced map \(f^*\) defined by \(f^*(uv) = \left\lfloor \frac{f(u) + f(v)}{2} \right\rfloor\) where \(\lfloor x \rfloor\) denote the least integer which is \(\leq x\) and \(|e_0(0) - e_1(1)| \leq 1\) where \(e_0(0)\) is no.of edges with zero label. \(e_1(1)\) is no.of edges with one label.

A graph that admits a mean cordial labeling is called a mean cordial graph. We proved that \(P_n \circ K_1\), \((P_n + K_1)\), \(P_n \times P_n\), \((P_n : C_3)\), \((P_n : S_1)\), \(P_n \times P_2\), \(P_n + 2K_1\), \(Z -(P_n)\) are mean cordial graphs.

Definition 2.1(Comb)
The **Corona** $G_1 \odot G_2$ of two graphs $G_1$ and $G_2$ is defined as the graph $G$ obtained by taking one copy of $G_1$ (which has $p_1$ points) and $p_1$ copies of $G_2$ and then joining the $i^{th}$ point of $G_1$ to every point in the $i^{th}$ copy of $G_2$. The graph $P_n \odot K_1$ is called a **comb**.

**Definition 2.2 (Fan)**

The **join** $G_1 + G_2$ of $G_1$ and $G_2$ consists of $G_1 \cup G_2$ and all lines joining $V_1$ with $V_2$ as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2) \cup \{ uv : u \in V(G_1) \text{ and } v \in V(G_2) \}$. The graph $P_n + K_1$ is called a **Fan**.

**Definition 2.3 (Grid)**

To define the **product** $G_1 \times G_2$, consider any two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V_1 \times V_2$. Then $u$ and $v$ are adjacent in $G_1 \times G_2$ whenever $(u_1 = v_1$ and $u_2 \text{ adj } v_2)$ or $(u_2 = v_2$ and $u_1 \text{ adj } v_1)$. The product $P_m \times P_n$ is called **planar grids**.

**Definition 2.4 ($P_n : C_3$)**

A vertex of cycle $C_3$ attached and every vertex of a path $P_n$ is denoted by $[P_n : C_3]$.

**Definition 2.5 ($P_n : S_1$)**

Star of length one is joined with every vertex of a path $P_n$ through an edge.

It is denoted by $[P_n : S_1]$.

**Definition 2.6 ([Z-(Pn)]**

In a pair of path $P_n$ $i^{th}$ vertex of a path $P_1$ is joined with $i+1^{th}$ vertex of a path $P_2$. It is denoted by $Z-(P_n)$.

**Definition 2.7 (Double fan)**

The **join** $G_1 + G_2$ of $G_1$ and $G_2$ consists of $G_1 \cup G_2$ and all lines joining $V_1$ with $V_2$ as vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2) \cup \{ uv : u \in V(G_1) \text{ and } v \in V(G_2) \}$. The graph $P_n + 2K_1$ is called the **Double fan**.

**Definition 2.8 (Ladder)**

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To define the \textit{product} $G_1 \times G_2$, consider any two vertices $u = (u_1, u_2)$ and $v = (v_1, v_2)$ in $V_1 \times V_2$. Then $u$ and $v$ are adjacent in $G_1 \times G_2$ whenever $(u_1 = v_1$ and $u_2$ adj $v_2$) or $(u_2 = v_2$ and $u_1$ adj $v_1$). The product $K_2 \times P_n$ is called \textit{Ladder}.

3. Main Results on Path Related Mean Cordial Graphs

\textbf{Theorem 3.1}

Graph $P_n \circ K_1$ is a Mean Cordial Graph.

\textbf{Proof:}

Let $G = (V, E)$

Let $G$ be $P_n \circ K_1$

Let $V[P_n \circ K_1] = \{(u_i, v_i): 1 \leq i \leq n\}$

Let $E[P_n \circ K_1] = \{(u_i, u_{i+1}): 1 \leq i \leq n-1\} \sqcup \{(u_i, v_i): 1 \leq i \leq n\}$

Define $f: V(G) \to \{0, 1, 2\}$ by

$f(u_i) = 1$

$f(v_i) = 0$

The induced edge labeling are

$f^*(u_i, u_{i+1}) = 1$

$f^*(u_i, v_i) = 0$

Hence the graph satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Therefore the graph $P_n \circ K_1$ is a mean cordial graph.
For example, the graph $P_4 \odot K_1$ is shown in the figure.

\begin{center}
\begin{tikzpicture}
  \node (v1) at (0,0) [circle,draw] {$v_1$};
  \node (v2) at (1,0) [circle,draw] {$v_2$};
  \node (v3) at (2,0) [circle,draw] {$v_3$};
  \node (v4) at (3,0) [circle,draw] {$v_4$};
  \node (u1) at (0,1) [circle,draw] {$u_1$};
  \node (u2) at (1,1) [circle,draw] {$u_2$};
  \node (u3) at (2,1) [circle,draw] {$u_3$};
  \node (u4) at (3,1) [circle,draw] {$u_4$};

  \draw (v1) -- (u1) node [midway, above] {$1$};
  \draw (v2) -- (u2) node [midway, above] {$1$};
  \draw (v3) -- (u3) node [midway, above] {$1$};
  \draw (v4) -- (u4) node [midway, above] {$1$};

  \draw (u1) -- (v1) node [midway, left] {$0$};
  \draw (u2) -- (v2) node [midway, left] {$0$};
  \draw (u3) -- (v3) node [midway, left] {$0$};
  \draw (u4) -- (v4) node [midway, left] {$0$};
\end{tikzpicture}
\end{center}

**Theorem 3.2**

Fan $(P_n + K_1)$ is a Mean Cordial Graph.

**Proof:**

Let $G = (V, E)$

Let $G$ be $(P_n + K_1)$

Let $V[P_n + K_1] = \{u, u_i : 1 \leq i \leq n\}$

Let $E[P_n + K_1] = \{(u_{i+1}) : 1 \leq i \leq n-1\}$

Define $f : V(G) \to \{0, 1, 2\}$ by

$f(u) = 2$

$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \text{mod } 2, 1 \leq i \leq n \\ 0 & \text{if } i \equiv 0 \text{mod } 2, 1 \leq i \leq n \end{cases}$

The induced edge labeling are

$f^*(u_{i+1}) = 1$

$f^*(u_i u_{i+1}) = 0$

Hence the graph satisfies the condition $|e_t(0) - e_t(1)| \leq 1$.

Therefore the graph $(P_n + K_1)$ is a mean cordial graph.
For example the graph \((P_2 + K_1)\) is shown in the figure

```
+---+---+---+
|1  |   | 0|
+---+---+---+
|2   |   |0  |
+---+---+---+
```

**Theorem 3.3**

Grid \(P_n \times P_n\) is a Mean Cordial Graph.

**Proof:**

Let \(G = (V, E)\)

Let \(G\) be \(P_n \times P_n\)

Let \(V(G) = \{u_{ij} : 1 \leq i \leq n\}\)

Let \(E(G) = \{(u_{ij} u_{(i+1)j}) : 1 \leq i \leq n, 1 \leq j \leq n-1\} \cup \{(u_{ij} u_{i(j+1)}) : 1 \leq i \leq n-1, 1 \leq i \leq n\}\)

Define \(f : V(G) \to \{0, 1, 2\}\) by

The vertex labeling are

When \(i \equiv 1 \mod 2\)

\(f(u_{ij}) = 1, 1 \leq i \leq n, 1 \leq j \leq n\)

When \(i \equiv 0 \mod 2\)

\(f(u_{ij}) = \begin{cases} 0 & \text{if } j \equiv 1 \mod 2, 1 \leq i \leq n \\ 1 & \text{if } j \equiv 0 \mod 2, 1 \leq j \leq n \end{cases}\)

The induced edge labelling are

\(f^*(u_{ij} u_{(i+1)j}) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n, 1 \leq j \leq n - 1 \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n, 1 \leq j \leq n - 1 \end{cases}\)

\(f^*(u_{ij} u_{i(j+1)}) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n - 1, 1 \leq j \leq n \\ 1 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n - 1, 1 \leq j \leq n \end{cases}\)

Here the graph satisfies the condition \(|e_f(0) - e_f(1)| \leq 1\).

Hence, \(P_n \times P_n\) is a mean cordial graph.
For example, the graph $P_4 \times P_4$ is shown in the figure.

\begin{center}
\includegraphics[width=\textwidth]{graph.png}
\end{center}

**Theorem 3.4**

Graph $(P_n : C_3)$ is a Mean Cordial Graph.

**Proof:**

Let $G = (V, E)$

Let $G$ be $(P_n : C_3)$

Let $V[G] = \{u_i : 1 \leq i \leq n, u_{ij} : 1 \leq i \leq n, 1 \leq j \leq 2\}$

Let $E[G] = \{(u_i, u_{i+1}): 1 \leq i \leq n\} \uplus \{(u_i, u_{1}): 1 \leq i \leq n\} \uplus \{(u_i, u_{2}): 1 \leq i \leq n\}$

Define $f : V(G) \rightarrow \{0, 1, 2\}$ by

\[
f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \end{cases}
\]

$f(u_{ii}) = 2, 1 \leq i \leq n$

$f(u_{i1}) = 0, 1 \leq i \leq n$

The induced edge labelling are

$f^*(u_i, u_{i1}) = 1, 1 \leq i \leq n$

$f^*(u_i, u_{i2}) = 0, 1 \leq i \leq n$
\[ f^*(u_i u_{i+1}) = 1, 1 \leq i \leq n \]
\[ f^*(u_i u_{i+1}) = 0, 1 \leq i \leq n \]

Hence, \( e_f(0) + 1 = e_f(1) \)

It satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \).

Hence, the graph \((P_n : C_3)\) is a mean cordial graph.

For example, the graph \((P_2 : C_3)\) is shown in the figure.

**Theorem 3.5**

Graph \([P_n : S_1]\) is a Mean Cordial Graph.

**Proof:**

Let \( G = (V, E) \)

Let \( G \) be \([P_n : S_1]\)

Let \( V[P_n : S_1] = \{u_i : 1 \leq i \leq n \}; (u_i) : 1 \leq i \leq n, 1 \leq j \leq 2\) \)

Let \( E[P_n : S_1] = \{(u_i, u_{i+1}) : 1 \leq i \leq n-1\} \square \{(u_i, u_{i+2}) : 1 \leq i \leq n\}\)

Define \( f : V(G) \rightarrow \{0, 1, 2\} \) by

\[ f(u_i u_{i+1}) = 1, 1 \leq i \leq n \]
\[ f(u_i) = 0, 1 \leq i \leq n \]
\[ f(u_{i+2}) = \begin{cases} 2 & \text{if } i \equiv 1 \mod{2} \\ 0 & \text{if } i \equiv 0 \mod{2} \end{cases}, 1 \leq i \leq n \]

The induced edge labeling are

\[ f^*(u_i u_{i+1}) = 1, 1 \leq i \leq n-1 \]
\[ f^*(u_{i_1} u_{i_2}) = 0, \ 1 \leq i \leq n \]

\[ f^*(u_{i_1} u_{i_2}) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2 \\ 0 & \text{if } i \equiv 0 \mod 2 \end{cases}, \ 1 \leq i \leq n \]

Here, \( e_f(1) + 1 = e_f(0) \), if \( n \) is even

\[ e_f(1) = e_f(0), \text{ if } n \text{ is odd} \]

It satisfies the condition \( |e_f(0) - e_f(1)| \leq 1 \).

Hence, \([P_n : S_1]\) is a mean cordial graph.

For example \([P_2 : S_1]\) and \([P_3 : S_1]\) are mean cordial graphs as shown in the figure.
**Theorem 3.6**

Ladder \([P_n \times P_2]\) is a Mean Cordial Graph.

**Proof:**

Let \(G = (V, E)\)

Let \(G = [P_n \times P_2]\)

Let \(V[P_n \times P_2] = \{(u_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2\}\)

Let \(E[P_n \times P_2] = \{(u_{ij} u_{i(i+1)_j}) : 1 \leq i \leq n-1, 1 \leq j \leq 2\}\)

Define \(f : V(G) \rightarrow \{0, 1, 2\}\) by

\[
f(u_{ij}) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n \\ 2 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \\
\end{cases}
\]

The induced edge labeling are

\[
f^*(u_{ij} u_{i(j+1)}) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n \\ 1 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \\
\end{cases}
\]

\[
f^*(u_{i1} u_{i+1}) = 1, 1 \leq i \leq n-1
\]

\[
f^*(u_{i2} u_{i+1}) = 2, 1 \leq i \leq n-1
\]

Hence, the graph satisfies the condition \(|e_f(0) - e_f(1)| \leq 1\).

Therefore, \([P_n \times P_2]\) is a mean cordial graph.

For example, the graph \([P_3 \times P_2]\) is shown in the figure.
Theorem 3.7

$Z - (P_n)$ is a Mean Cordial Graph.

Proof:

Let $G = (V, E)$

Let $G$ be $Z - P_n$

Let $V[Z - P_n] = \{u_i, v_i : 1 \leq i \leq n\}$

Let $E[Z - P_n] = \{[(u_i, u_{i+1}) \in E(G)] \cup [(v_i, v_{i+1}) \in E(G)] : 1 \leq i \leq n-1\}$

Define $f : V(G) \rightarrow \{0, 1, 2\}$ by

$f(u_i) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \end{cases}$

$f(v_i) = \begin{cases} 2 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n \end{cases}$

The induced edge labeling are

$f^*(u_i, u_{i+1}) = 0, 1 \leq i \leq n-1$

$f^*(v_i, v_{i+1}) = 1, 1 \leq i \leq n-1$

$f^*(v_i, u_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \mod 2, 1 \leq i \leq n-1 \\ 0 & \text{if } i \equiv 0 \mod 2, 1 \leq i \leq n-1 \end{cases}$

Here, $e_f(0) = e_f(1)$ when $n$ is odd.

$e_f(0) + 1 = e_f(1)$ when $n$ is even.

It satisfies the condition $|e_f(0) - e_f(1)| \leq 1$.

Hence, $Z - (P_n)$ is a mean cordial graph.

For example, the graphs $Z - (P_4)$ and $Z - (P_3)$ are shown in the figure.
Theorem 3.8

Double Fan \( P_n + 2K_1 \) is a Mean Cordial Graph.

Proof:

Let \( G = (V, E) \)

Let \( G \) be \( P_n + 2K_1 \)

Let \( V[P_n + 2K_1] = \{ u, v, u_i : 1 \leq i \leq n \} \)

Let \( E[P_n + 2K_1] = \{ (u v_i) \setminus (v u_i) : 1 \leq i \leq n \} \setminus \{ (u_i u_{i+1}) : 1 \leq i \leq n-1 \} \)

Define \( f : V(G) \to \{0, 1, 2\} \) by

\[ f(u) = 1 \]

\[ f(v) = 2 \]
f(u_i) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2, \ 1 & \text{if } i \equiv 0 \mod 2 \end{cases}, 1 \leq i \leq n

The induced edge labeling are

f^*(u_{u_i}) = \begin{cases} 0 & \text{if } i \equiv 1 \mod 2, \ 1 & \text{if } i \equiv 0 \mod 2 \end{cases}, 1 \leq i \leq n

f^*(v_{u_i}) = 1, 1 \leq i \leq n

f^*(u_i, u_{i+1}) = 0, 1 \leq i \leq n-1

Here e_0(0) = e_i(1) when n is odd.

e_0(0) + 1 = e_i(1) when n is even.

Hence it satisfies the condition |e_0(0) - e_i(1)| \leq 1.

Therefore the graph P_n + 2K_1 is a mean cordial graph.

For example P_2 + 2K_1 and P_3 + 3K_1 are shown in the figure.

REFERENCES


