Higher Dimensional Universe in Scalar-Tensor Theory

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Abstract

We consider the Kaluza-Klein type cosmological model in scalar-tensor theory proposed by Lau and Prokhovnik. The exact solution of the field equations are obtained by using the gamma law equation of state, $p = (\gamma - 1)\rho$ in which the parameter $\gamma$ depends on scale factor $R$. The functional form of $\gamma(R)$ is used to describe a wide range of cosmological solution at early universe for two phases in cosmic history: inflationary phase and radiation dominated phase. The solutions show the power law expansion and cosmological constant is found to be positive and decreasing function of cosmic time. The solutions are compatible with the Dirac’s large number hypothesis. The deceleration parameter has been presented in a unified manner in terms of scale factor which describes the inflation of the model.

Key-words: Early universe, Kaluza-klein cosmological model, Inflationary phase, Radiation dominated phase.

1. Introduction

The study of higher-dimensional space time theories is becoming most interesting research for a unification of all fundamental interactions, including gravitation in the frame work of a theory of general relativity in higher dimension. The most famous five dimensional theory proposed by Kaluza[1] and Klein[2] was the first in which gravitation and electromagnetism could be unified in a single geometrical structure. Overduin and Wesson[3] have discussed an excellent review of Kaluza-Klein theory and higher dimensional unified theories in which the cosmological and astrophysical implications of extra dimension have been studied. Also many authors have studied Kaluza-Klein cosmological models with different matters [4-8]. There is now extensive literature dealing with different aspect of higher dimensional cosmologies. Some authors explain entropy production [9, 10] and inflation [11, 12] as a result of contraction of extra space. Many cosmological solutions of Einstein field’s equations with different equation of state and different symmetries, in presence or not of a cosmological constant, has been found with 5D [13-15] and also with arbitrary number of dimension [16-21]. Dirac [22] proposed a theory with variable $G$ motivated by the numerology uncovered by Weyl, Eddington and Dirac himself. Dirac put the argument through his large numbers hypothesis (LNH), which states that any two of the very large numbers occurring in nature are connected by a simple mathematical relation in which the coefficient are of the order of unity. Several attempts have been made to formulate a generalized field theory of gravitation in which $G$ is a scalar function of coordinates and Einstein’s theory of gravitation appears as a special case of the new theory. Jordan [23] studied a five-dimensional theory of relativity,
where $G$ is generalized to be a scalar function of the age of the universe and the covariant divergence of the energy-momentum tensor is non-zero. Dirac himself [24, 25] studied the ‘two metrics’ theory where two unit systems are set up, namely the Einstein and the atomic units. Lau [26] proposed a theory by considering Einstein’s field equations with a non-zero cosmological term $\Lambda$, with $\Lambda$ and $G$ assuming time-dependent forms in the field equations in order to satisfy the LNH without needing the ‘two metrics’ theory. Motivated by this conjecture and the LNH, Lau and Prokhovnik [27] generalized Lau’s theory by formulating a new scalar-tensor theory in terms of an action principle. This is a theory with variable cosmological constant and gravitational constant but, in addition, it has a scalar field $\Psi$. The theory developed is applied to a cosmological model compatible with the Dirac’s Large Number Hypothesis. A time dependent scalar potential $\Psi = \Psi(t)$ is introduced such that $\Lambda = \Lambda(\Psi)$ and $G = G(\Psi)$ are coupled. This theory was further investigated by Beesham [28], Maharaj and Beesham [29] for the flat Robertson-Walker model. Maharaj and Naidoo [30] studied Robertson-Walker model in Lau-Prokhovnik’s theory by imposing the constant deceleration parameter. Nowadays, the parameter $\Lambda$ is believed to correspond to the vacuum energy density of the quantum field [31], and it is thought that $\Lambda$ was large during the early stages of the evolution of the universe. The status of the cosmological constant problem was reviewed by Weinberg [32]. It is worth noting that cosmological models with varying $\Lambda$ have been the subjects of numerous papers in recent years. Bertolami [33, 34], Ratra and Peebles [35], Carvalho et al. [36] and Berman [38, 39] have worked out cosmologies based on time-dependent $\Lambda$. Overduin and Cooperstock [40], and Arbab [41] studied cosmological models in a more phenomenological way by assuming some functional forms of $\Lambda$. Singh, C.P. and Beesham [42] study early universe in scalar-tensor theory. Khadekar, et al [43] also discussed early universe with variable cosmological constants in higher dimensional space-time. In cosmology, the evolution of universe is described by Einstein’s equations of gravity together with an equation of state for the matter content. Usually the field equations are solved and analyzed separately for the different epochs, although some authors have given unified solutions. Israelit and Rosen [44] used a different equation of state in which the pressure varies continuously from $(p = -\rho)$ to its value during the radiation era $(p = \frac{\rho}{3})$ and then radiation to matter-dominated era $(p = 0)$. Carvalho [37] presented a similar type of work by considering a model as it goes from an inflationary phase to a radiation-dominated era. He studied the model by using “gamma-law” equation of state but the adiabatic parameter ‘gamma’ varies with cosmic time as the universe expands.

In this paper, we extend Carvalho’s work in a scalar tensor theory proposed by Lau and Prokhovnik. We study the evolution of early universe for Kaluza-Klein type cosmological model as it goes from an inflationary phase to radiation-dominated era. The paper is presented as follows: we discussed model and field equations in Sect. 2 and solutions of field equations in Sect. 3. The conclusions are discussed in Sect. 4.

2. Model and Field Equations.

We consider the Kaluza-Klein type metric

$$ds^2 = dt^2 - R^2(t)(dx^2 + dy^2 + dz^2) - B^2(t)d\Phi^2$$

(1)
where $B(t)$ and $R(t)$ are the scale factors. The Universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of a perfect fluid

$$T_{\mu\nu} = (\rho + p)U_{\mu}U_{\nu} - pg_{\mu\nu}$$  \hspace{1cm} (2)$$

where $\rho$ is the energy density of the cosmic matter and $p$ is its pressure and $u_{\mu}$ is the five velocity vector such that $u_{\mu}u^{\mu} = 1$.

Using a variation action principle, Lau and Prokhovni[27] proposed the generalized field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu} + \psi_{,\mu}\psi_{,\nu}$$  \hspace{1cm} (3)$$

where commas denote partial differentiation with respect to cosmic time, $R_{\mu\nu}$ is the Ricci tensor, $R = R^{\mu}_{\mu}$ is the Ricci scalar, $T_{\mu\nu}$ is the energy momentum tensor and $g_{\mu\nu}$ is the metric tensor. The cosmological parameter $\Lambda$ and gravitational constant $G$ are functions of the scalar function $\psi$. The field equations (3) is a generalization of the classical Einstein’s field equations and Lau’s[26] theory to incorporate variables cosmological parameter $\Lambda$ and gravitational parameter $G$. The quantity

$$\Lambda = \lambda(t) - \frac{1}{2}g^{\mu\nu}\dot{\psi}^2$$  \hspace{1cm} (4)$$

is a generalization of the usual cosmological constant. Here overdots denote differentiation with respect to cosmic time, $\Lambda$ is the cosmological constant used in variation of action principle by Lau and Prokhovnik. The scalar potential $\psi$ is strictly time dependent and couples $\Lambda$ and $G$.

$$\psi = \psi(t), \Lambda = \Lambda(\psi), G = G(\psi)$$  \hspace{1cm} (5)$$

In general, $\psi$ and $\Lambda$ could be functions of time and space, but since we will be considering only the spatially homogeneous Kaluza-Klein type cosmological model in what follows, we take them to functions of time alone. Coupled to (3), the field equation for $\psi$ is

$$\ddot{\psi} + \Box\psi + \Lambda + \frac{1}{2}g^{\mu\nu}\dot{\psi}^2 + g^{\mu\nu}\dot{\psi}\dddot{\psi} + 8\pi GL_m = 0$$  \hspace{1cm} (6)$$

where $L_m$ is the matter Lagrangian density equal to the matter energy density including all non-gravitational fields and

$$\Box \psi = g^{\mu\nu}\psi_{,\mu\nu}$$  \hspace{1cm} (7)$$

where semi-colons denote covariant derivative with respect to cosmic time.

Using co-moving co-ordinates $u_{\mu} = (1, 0, 0, 0, 0)$ in (2) and with line element (1), Einstein’s field (3) by assuming $B(t) = R^n$ yields three independent equations

$$3(n + 1)\frac{\dot{R}^2}{R^2} - \Lambda(t) = 8\pi G(t)\rho + \Psi(t)^2$$  \hspace{1cm} (8)$$

$$\frac{n + 2}{R} \frac{\ddot{R}}{R} + (n^2 + n + 1)\frac{\dot{R}^2}{R^2} = -8\pi G(t) + \Lambda(t)$$  \hspace{1cm} (9)$$

$$3\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2}\right) = -8\pi G(t)p + \Lambda(t)$$  \hspace{1cm} (10)$$
The field equations (8)-(10) can be written as

$$3(n+1)\left(\frac{\dot{R}}{R} + \frac{\dot{R}^2}{R^2}\right) = -8\pi G \left[(n+1)p + \rho - \frac{n\Lambda}{8\pi G}\right]$$  \hspace{1cm} (11)

$$3(n+1)\frac{\dot{R}^2}{R^2} = 8\pi G(t) \left[\rho + \frac{\Lambda}{8\pi G} + \frac{\Psi(t)^2}{8\pi G}\right]$$  \hspace{1cm} (12)

Using the metric (1), it is easy to show that the field equations (6) reduces to

$$2\dot{\Psi}^2 + \dot{\Lambda} + 3(n+1)\frac{\dot{R}}{R} \dot{\Psi} + 8\pi G\rho = 0$$  \hspace{1cm} (13)

The usual energy momentum conservation relation $T_{\mu\nu}^{\mu\nu} = 0$ leads to Equations (12), (13) and (14) are a system of three equations for six unknowns $R$, $\rho$, $p$, $G$, $\Lambda$ and $\Psi$. In order to solve the system of equations (12), (13) and (14), we first assume the relation between pressure and energy density through the "gamma-law" equation of state

$$p = (\gamma - 1)\rho$$  \hspace{1cm} (14)

where $\gamma$ is the adiabatic parameter. In general, the value of $\gamma$ is taken to be constant lying between $0 \leq \gamma \leq 2$ for different epochs. Our aim in this paper is to study how the adiabatic parameter should vary so that in the course of its evolution the universe goes through a transition from an inflationary phase to a radiation-dominated phase. Carvalho $^{37}$ assumed a scale dependent $\gamma(R)$ of the form

$$\gamma(R) = \frac{4}{3} \frac{A(R/R_\ast)^2 + (a/2)(R/R_\ast)^a}{A(R/R_\ast)^2 + (R/R_\ast)^a}$$  \hspace{1cm} (15)

where $A$ is a constant, $a$ is a free parameter related to the power of the cosmic time $t$ during the inflationary era, and for $a \to 0$, we have an exponential inflation ($\gamma = 0$ at $R=0$) and $R_\ast$ is a certain reference value of scale factor $R$. The above functional form of $\gamma$ is an increasing function of $R$. In the limit $R \to 0$, we have

$$\gamma(R) = \frac{2a}{3}$$  \hspace{1cm} (16)

The expression for $\gamma$ in (16) expresses the evolution of the universe as it goes through a transition from an inflationary phase to a radiation-dominated phase. The function $\gamma(R)$ is such that when the scale factor $R$ is less than a certain reference value $R_\ast (R \ll R_\ast)$, we have the inflationary phase and when the scale factor is greater than $R_\ast (R \gg R_\ast)$, we have radiation-dominated phase. We assume the cosmological parameter $\Lambda$ and scalar field $\Psi$ as

$$\Lambda = 3\alpha H^2$$  \hspace{1cm} (17)

$$\Psi = \alpha_1 \ln R + \alpha_o$$  \hspace{1cm} (18)

where $\alpha$, $\alpha_1$ and $\alpha_o$ are all positive constants and $H = \frac{\dot{R}}{R}$ is the Hubble parameter.

The ansatz [17] initially has been proposed by Carvalho et al.[36] on the dimensional ground, widely used to study decaying vacuum cosmological models (Overduin and
Cooperstock[40] and Arbab[41]. The form of $\Psi$ in (18) is the simplest nonlinear one, which is compatible with the Dirac’s LNH. Using (17) and (18) in (12) and simplifying, we obtain

$$2\frac{H'}{H} = \frac{\rho'}{\rho} + \frac{G'}{G}$$

(19)

where a prime denotes derivative with respect to scale factor. Using (15), (14) can be written as

$$\frac{\rho'}{\rho} = -\frac{3\gamma(R)}{R}$$

(20)

Again, using (17) and (18), (13) can be written as

$$2\alpha_1^2 + 6\alpha + \frac{H'}{H} = \frac{2\alpha_1^2 + (3 + n)}{H} + \frac{G'}{G} - \frac{\gamma(R)}{R} = 0$$

(21)

Using (19) and (20) in (21), we finally obtain

$$\frac{H'}{H} + \left[\frac{(3 + n)(n + 1)}{6(n + 1)} + \frac{3(n + 1) - 3\alpha - \alpha_1^2}{6(n + 1)}\right] \gamma(R) \frac{1}{R} = 0$$

(22)

and its integral is

$$H = D \exp \left[\frac{-3}{2} \frac{1 + n/3}{n + 1} \int \frac{\gamma(R)}{R} dR - \frac{(3 + n)\alpha_1^2}{6(n + 1)} \int \frac{1}{R} dR\right]$$

(23)

3. Solution of the Field Equations.

Using the value of $\gamma(R)$ from (16) into (23) and integrating, the Hubble parameter is given by

$$H = \frac{D}{R \left(\frac{(3 + n)\alpha_1^2}{6(n + 1)} \left(A(R/R_*)^2 + (R/R_*)^a\right) \right) - \frac{(n + 1 - \alpha - \alpha_1^2/3)(1 + n/3)}{n + 1}}$$

(24)

where D is the constant of integration. If $H = H_*$ for $R = R_*$, we have a relation between constant A and D, is given by

$$D = H_* R_* \frac{(3 + n)\alpha_1^2}{6(n + 1)} \frac{\left(A(R/R_*)^2 + (R/R_*)^a\right) \left(n + 1 - \alpha - \alpha_1^2/3\right)}{n + 1}$$

(25)

Using this value of D, we have that the Hubble parameter is given by

$$H = \frac{H_* R_* \frac{(3 + n)\alpha_1^2}{6(n + 1)} \left(1 + A\right) \frac{\left(n + 1 - \alpha - \alpha_1^2/3\right)(1 + n/3)}{n + 1}}{R \left(\frac{(3 + n)\alpha_1^2}{6(n + 1)} \left(A(R/R_*)^2 + (R/R_*)^a\right) \right) - \frac{(n + 1 - \alpha - \alpha_1^2/3)(1 + n/3)}{n + 1}}$$

(26)

Now we study (26) for two early phases of evolution of universe-inflationary and radiation-dominated phases.

For inflationary phase($R \ll R_*$), the second term in the right-hand side of denominator of (26) dominates and one has a phase of power-law inflation which is given by

$$R = R_* \left[H_* \left(1 + \frac{\left(n + 1 - \alpha - \alpha_1^2/3\right)(1 + n/3)}{n + 1}\right) \frac{1}{B_0} t\right]^{B_0}$$

(27)
where $B_o = \frac{6(n+1)}{(n+3)\alpha_1^2+6a\left[(n+1-\alpha-\alpha_1^2/3)(1+n/3)\right]}$

Equation (27) shows that during inflation, the dimensions of the universe increase according to law

$$R \propto t^{B_o}$$  \hspace{1cm} (28)

The Hubble parameter in terms of cosmic time $t$ is given by

$$H = B_o t^{-1}$$  \hspace{1cm} (29)

The other physical parameters have the following expressions:

$$\Lambda = 3\alpha B_o t^{-2}$$  \hspace{1cm} (30)

$$\Psi(t) = \alpha_1 B_o \log t + \alpha_{oo}$$  \hspace{1cm} (31)

where $\alpha_{oo}$ is the constant of integration. The gravitational parameter has the form:

$$G \propto t^{-b}$$  \hspace{1cm} (32)

where $b = \frac{6(n+1)(n+3)\alpha_1^2-(2\alpha_1^2+6a)\left[(3+n)\alpha_1^2+6a(n1-\alpha-\alpha_1^2/3)(1+n/3)\right]}{(n+3)\alpha_1^2+6a(n1-\alpha-\alpha_1^2/3)(1+n/3)\left[3(n+1)-3\alpha-3a_1^2\right]}$

The energy density has the form

$$\rho \propto t^{-(2-b)}$$  \hspace{1cm} (33)

Equation (27) shows that during the power-law inflation, the dimensions of the universe increase according to law $R \propto t^{B_o}$ and for expanding universe we must have $\frac{6(n+1)}{(n+3)\alpha_1^2+6a\left[(n+1-\alpha-\alpha_1^2/3)(1+n/3)\right]} > 0$. In addition, the constants have to be chosen so that $\dot{R}$ is positive. Equation (27) also indicates that $R = 0$ at $t = 0$, $R \to \infty$, and $R \to 0$.

For physical significance $\rho > 0$, we must have the positive proportionality constant. The energy density and gravitational parameter are decreasing function of cosmic time in $0 \leq b \leq 2$. Compatibility with Dirac LNH can be achieved if $b = 1$. The cosmological parameter varies as the inverse square of the cosmic time. This form of $\Lambda$ is physically reasonable as observations suggest that $\Lambda$ is very small in the present-day universe. It will approach zero when the age of the universe tends to infinity; otherwise, $\Lambda$ is non-zero as a consequence of LNH. According to the recent cosmological observations (Perlmutter et al.\[45\], Riess et al.\[46\]), the universe is presently accelerating. A positive $\Lambda$ causes an acceleration in the expansion of the universe, where as a negative $\Lambda$ decelerates the expansion. We observe that $\dot{\psi} \propto t^{-1}$ which shows that $\dot{\psi} \to 0$ as the age of the universe increases indefinitely. We also observe that the rate of decrease of the gravitational parameter is given by

$$\frac{\dot{G}}{G} = t^{-1}$$  \hspace{1cm} (34)

Therefore, under most circumstances, $G$ can be regarded as a genuine constant after a time, which is distant enough from Big Bang. The implication of a time varying $G$ will take effect only in the early epoch. We also note that $\left|\frac{\dot{A}}{A}\right| = t^{-1}$. As in the case
of G, Λ can be treated as a genuine constant after the early epoch. Hence, with the approximation of zero Λ and constant G, we can immediately go over to Einstein’s field equations.

For radiation-dominated phase \((R \gg R_*)\), we have from (27)

\[
R = R_* \left[ H_* \left( \frac{1 + A}{B_1} \right) \frac{(n+1-\alpha - \alpha_1^2/3)(1+n/3)}{n+1} t \right]^{B_1} \quad (35)
\]

where

\[
B_1 = \frac{6(n+1)}{(n+3)\alpha_1^2 + 12 \left[ (n+1-\alpha - \alpha_1^2/3)(1+n/3) \right]}
\]

The Hubble parameter, cosmological parameter and scalar field in terms of cosmic time are respectively given by

\[
H = B_1 t^{-1} \quad (36)
\]

\[
\Lambda = 3\alpha B_1^2 t^{-2} \quad (37)
\]

\[
\Psi(t) = \alpha_1 B_1 \ln t + \alpha_{ooo} \quad (38)
\]

where \(\alpha_{ooo}\) is the constant of integration. The gravitational parameter has the form

\[
G \propto t^{-b_1} \quad (39)
\]

where

\[
b_1 = \frac{6(n+1)(n+3)\alpha_1^2 - (2\alpha_1^2 + 6\alpha) \left[ (3+n)\alpha_1^2 + 12(n+1-\alpha - \alpha_1^2/3)(1+n/3) \right]}{(n+3)\alpha_1^2 + 12(n+1-\alpha - \alpha_1^2/3)(1+n/3) \left[ 3(n+1) - 3\alpha - 3\alpha_1^2 \right]}
\]

The energy density has the form

\[
\rho \propto t^{-(2-b_1)} \quad (40)
\]

The physical interpretations in the case of radiation-dominated phase are same as discussed in inflationary phase. Since \(\Lambda \propto t^{-2}\) the magnitude of \(\Lambda\) will decrease as the age of the universe increases. For the compatibility with Dirac’s LNH \((R \propto t^{1/3})\), we have

\[
\frac{6(n + 1)(2n - 3)}{(3 + n)} = \alpha_1^2 + 12\alpha \quad (41)
\]

From (26), a unified expression for deceleration parameter can be given in terms of scale factor as

\[
q = \frac{\left[ \frac{2(1+n/3)(n+1-\alpha - \alpha_1^2/3)}{3(n+1)} + \frac{(3+n)\alpha_1^2}{6(n+1)} - 1 \right] A(R/R_*^2)}{A(R/R_*^2) + (R/R_*)^a} + \frac{\left[ \frac{a(1+n/3)(n+1-\alpha - \alpha_1^2/3)}{3(n+1)} + \frac{(3+n)\alpha_1^2}{6(n+1)} - 1 \right] A(R/R_*^a)}{A(R/R_*^2) + (R/R_*)^a} \quad (42)
\]

Therefore, \(q\) varies from

\[
\left[ \frac{2(1+n/3)(n+1-\alpha - \alpha_1^2/3)}{3(n+1)} + \frac{(3+n)\alpha_1^2}{6(n+1)} - 1 \right] \text{ for } (R \ll R_*)\]

to

\[
\left[ \frac{a(1+n/3)(n+1-\alpha - \alpha_1^2/3)}{3(n+1)} + \frac{(3+n)\alpha_1^2}{6(n+1)} - 1 \right] \text{ for radiation phase. The sign of the deceleration parameter indicates whether the model inflates or not. The negative value of } q \text{ accelerates}
\]
the universe where as positive value decelerates the universe. We now study the solution in the limit $a \to 0$. In this case, (26) becomes

$$H = \frac{H_0 R_{*}^{\frac{(3+n)a_1^2}{2(n+1)}}}{R^{\frac{6(n+1)}{3(n+1)}}} \left[ (1 + A) \frac{(n+1-a_1^2/3)(1+n/3)}{n+1} \right]^{(n+1-a_1^2)/(1+n/3)}(43)$$

In the limit of very small $R$, the second term on the right hand side in denominator of (43) dominates and one has power-law inflation, which is given

$$R = \left[ H_0 R_{*}^{\frac{(3+n)a_1^2}{6(n+1)}} (1 + A) \frac{(n+1-a_1^2/3)(1+n/3)}{n+1} \right]^{(3+n)a_1^2/6(n+1)}(44)$$

Again, the radiation phase is described by the same solution as obtained in (35). We observe that for $a = 0$, the universe is finitely old, since $R = 0$ as $t \to 0$. There is the singularity since the energy density assumes infinite value as $R \to 0$. It can be inferred from (12) and (43) that for $R = 0$, $\rho \to \infty$. The deceleration parameter has the following form in terms of $R$

$$q = \left[ \frac{2(1+n/3)(n+1-a_1^2/3)}{3(n+1)} + \frac{(3+n)a_1^2}{6(n+1)} - 1 \right] A(R/R_*)^2 + 1 \frac{(3+n)a_1^2}{6(n+1)} - 1$$

and varies from $q = \frac{(3+n)a_1^2}{6(n+1)} - 1$ at $R = 0$ to $q = \left[ \frac{2(1+n/3)(n+1-a_1^2/3)}{3(n+1)} + \frac{(3+n)a_1^2}{6(n+1)} - 1 \right]$ for $R \gg R_*$ as expected.

4. CONCLUSIONS

The existence of the extra dimension is a general feature in theories beyond the standard model in particle physics. It may manifest itself as source of energy in the ordinary three spaces, such as "effective" dark energy or even "effective" dark matter. The geometrical structure and evolution pattern of extra dimension therefore may play an important role in cosmology. It has been suggested that higher dimensional cosmologies play an important role in the very early universe (Chodos [47]). A solution of the Einstein field equations with variable $\Lambda$ and $G$ in the context of higher dimensional space time is presented in this work. In this paper, we have discussed Kaluza-Klein type cosmological model filled with perfect fluid in scalar-tensor theory proposed by Lau and Prokhovnik.

We have solved the generalized field equations for two early phases of evolution of universe by using variation of "gamma-law" equation of state in which the adiabatic parameter ‘gamma’ varies continuously as universe expands. For $a = 0$, the parameter $\gamma$ varies from 0 for $R = 0$ to $4/3$ when $R \gg R_*$. The universe is finitely old and has power-law inflation. There is a big bang singularity since the energy density diverges as $R \to 0$, which gives a similar result as obtained by Carvalho in General Relativity Theory. We have assumed that the cosmological parameter is a quadratic function of the Hubble parameter and that the scalar field is a logarithmic function of the scale factor. For $a$ in the range $0 < a < 1$, $\gamma$ slowly increases from $2a/3$ for inflationary phase to radiation-dominated phase $4/3$. The first period of evolution is described by inflationary phase
with power-law inflation. This is then followed by a radiation-dominated era with power-law expansion. The cosmological parameter is a decreasing function of time, while the gravitational parameter tends to a constant value at late times. The implication of time varying $G$ will take effect only in the early epoch. The possibility that the cosmological parameter and gravitational parameter are not real constant is an intriguing possibility, which has intensively been investigated in the physical literature. It is a very plausible hypothesis that these effects were much stronger in the early universe. The solution found that $\Lambda$ - term varies inversely as square of the cosmic time. Therefore, our solution is consistent with the observation of the present-day values of the cosmological constant which are very small. Since $\rho$ and $G$ varies inversely with time, $\rho$ and $G$ decrease as the age of the universe increases. It is observed that the universe expands so ever with cosmic time starting from zero volume. The possibility of a positive $\Lambda$ - term and negative deceleration parameter is obtained in our results, which indicates that the expansion of the present universe is accelerating. Dirac’s LNH is found to be compatible with the result obtained. The results show a generalization of Carvalho’s work to a simple scalar-tensor theory. Thus, we have showed that by using a varying adiabatic parameter, it is possible to describe the two early phases of evolution of the universe in a unified manner. The solutions obtained in the present paper could give an appropriate description of the early period of our universe as expected. We are also studying whether a variable gamma-law equation of state could explain the continuous transition from a decelerated universe to an accelerated one, as is presently observed.

**References**

Math. Kl., 966 (1921)


