LAMINAR BOUNDARY LAYER FLOW OF NON-NEWTONIAN POWER LAW FLUID PAST A CONTINUOUSLY MOVING POROUS FLAT PLATE WITH SUCTION/INJECTION

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Abstract: - Laminar boundary layer flow of Non-Newtonian Power law fluid past a continuously moving porous flat plate has been considered. The governing equations of continuity and momentum are transformed into ordinary differential equations using similarity transformations. The equations are solved by using method of successive approximations starting with zeroth approximation. For \( n=1 \) the results tallies with Corresponding results for Newtonian fluids. Velocity profiles have been drawn for different values of parameter \( n \); suction/injection parameter \( f_w \) shows the behavior of power law fluids.

Keywords: - boundary layer, power law fluids, successive approximations, suction/injection parameter, velocity profiles.

Introduction: Boundary layer flows of viscous, incompressible fluid past semi infinite flat plate were studied by Blassius [1], Howarth [2]. Tsou et.al [3] studied flow and heat transfer in the boundary layer flow on a continuously moving flat surface. Mahmoud and Mahmoud [4] had given the analytical solutions of hydro magnetic boundary-layer flow of a non Newtonian power law fluid past continuously moving flat surface. Anuar Ishak [5] have investigated the steady boundary-layer flow of a non-Newtonian power-law fluid over a flat plate in a moving fluid. Approximate solutions to laminar boundary layer flow of power law fluid past a flat plate were obtained by Jadhav & Wagmode [6]. They have used the method of successive approximations starting with zeroth approximation to find the solution of the problem. T.G.Mayer[7], Chabra et al[8] have obtained an approximate analytical solution to power law fluid past flat plate. Gabriella Bognár, Zoltán Csáti[9] have obtained numerical solution to Boundary Layer Problems over Moving Flat Plate in Non-Newtonian Media.

WA Khan et.al [10] have studied Non –Newtonian flow of power law fluid over continuously moving flat plate in porous medium. In this paper we have extended the problem of Jadhav & Wagmode [6] to Non-Newtonian Power law fluid flow past continuously moving flat porous plate with suction /injection effects.

Mathematical Analysis:

Consider a steady, two dimensional flow of an electrically conducting, non-Newtonian power law fluid past a continuously moving flat porous plate. The \( x \)-axis is taken along the direction of flow and \( y \)-axis normal to it. The plate is assumed to be moving with uniform velocity \( U \). The governing boundary layer equations are

\[
\frac{u}{\partial x} + v \frac{\partial u}{\partial y} = -\gamma \frac{\partial}{\partial y} \left( -\frac{\partial w}{\partial y} \right) \tag{1}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{2}
\]

With boundary conditions

\( u = U, \ \nu = \nu_0(x) \) at \( y=0 \) and \( u \to 0, \nu \to 0 \) as \( y \to \infty \) \( \tag{3} \)
Where \( v_0(x) \) is suction velocity.

Introduce a stream function \( \Phi(x,y) \) such that \( u = \frac{\partial \Phi}{\partial y}, \ v = -\frac{\partial \Phi}{\partial x} \) -----(4)

Substituting equation (4) in (1), it reduces to

\[
\frac{\partial^2 \Phi}{\partial y^2} \frac{\partial^2 \Phi}{\partial x^2} - \frac{\partial^2 \Phi}{\partial x \partial y} = -\gamma \frac{\partial}{\partial y} \left( -\frac{\partial^2 \Phi}{\partial y^2} \right)^n \quad -----(5)
\]

Let \( \eta = y \left( \frac{U^{2n}}{\gamma x} \right)^{\frac{1}{n+1}} \) , \( \Phi(\eta) = (\gamma x U^{2n-1})^{\frac{1}{n+1}} f(\eta) \) -----(6)

Substituting the values in (6) to the equation (5) reduces to

\[n(n+1)(-f'')^{n-1}f''' - ff'' = 0 \quad -----(7)\]

\[f''' - \frac{1}{n(n+1)} f (f'')^{2-n} = 0 \quad -----(8)\]

The Boundary conditions are

\[f(0) = \frac{(n+1)v_0(x)(R_0)^{1/2}}{u} = f_w, \ f'(0) = 1, f'(\infty) = 0 \quad -----(9)\]

Where \( f_w > 0 \) for suction and \( f_w < 0 \) for injection.

METHOD OF SOLUTION:

To solve the non-linear differential equation (8) under the boundary conditions (9), we use method of successive approximations starting with zeroth approximation.

For zeroth approximation, we assume

\[f(\eta) = f_w + \frac{1}{\beta} - \frac{1}{\beta} e^{-\beta \eta} \quad -----(10)\]

Where \( \beta \) is arbitrary constant to be determined such that for the first approximation \( f'(0) = \varepsilon \), i.e., \( \beta \) is real root of the equation

\[\beta^{n+1} \left( \frac{1+\beta \eta}{n(n+1)(n-2)^2} - \frac{1}{n(n+1)(n-2)^2} \right) = 0 \quad -----(11)\]

The different successive approximations can be obtained from

\[f''' = \frac{1}{n(n+1)} [ f_{i-1}(-f_{i-1}''\eta^{2-n}) ] \quad -----(12)\]

For the first approximation, we have,

\[f_1''' = \frac{1}{n(n+1)} [ f_0(-f_0''\eta^{2-n}) ] \quad -----(13)\]

Integrating (13) with boundary conditions (9), we obtain

\[f_1(\eta) = A_1 e^{(n-2)\beta \eta} - A_2 e^{(n-3)\beta \eta} \quad -----(14)\]

Where,

\[A_1 = \frac{(1+\beta f_w)}{n(n+1)(n-2)^2 \beta^{n+2}}, \quad A_2 = \frac{1}{n(n+1)(n-3)^2 \beta^{n+1}} \quad -----(15)\]

Values of \( \beta \) can be obtained for various values of power index \( n \) and suction/injection parameter \( f_w \).
For different values of $\beta$, power index $n$, suction/injection parameter $f_w$, values of $f'_1(\eta)$ can be obtained. Hence Velocity profiles can be drawn for various values of $n$ and suction/injection parameter $f_w$.

For $f_w = 0$, the solution of the equation (12) agrees with the solution obtained by Jadhav and Wagmode [6] for flat plate. For $n=1$ the results tallies with the corresponding results for Newtonian fluids.

For different values of $n$ and suction/injection parameter $f_w$, skin friction coefficient $c_f^* = [−f'_1(0)]^n$ are obtained

<table>
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<tr>
<th>$f_w/n$</th>
<th>0.5</th>
<th>0.8</th>
<th>1.0</th>
<th>1.5</th>
<th>1.8</th>
</tr>
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<tr>
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<td>0.7820</td>
<td>0.5719</td>
<td>04085</td>
<td>0.2149</td>
<td>0.1178</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8311</td>
<td>0.6277</td>
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<td>0.2712</td>
<td>0.1584</td>
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<tr>
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<tr>
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<td>0.3534</td>
<td>0.1083</td>
<td>0.0081</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

**DISCUSSIONS:**

From the table, it is seen that as suction ($f_w > 0$) increases then skin friction $c_f^*$ increases and as injection ($f_w < 0$) then skin friction $c_f^*$ decreases for fixed value of power law index $n$.

For fixed value of suction /injection parameter $f_w$ as power law index increases then skin friction $c_f^*$ decreases as $n$ increases.

For different values of power law index $n$ and suction /injection parameter $f_w$, velocity profiles have been drawn shows the behavior of the power law fluids. For fixed values of suction /injection parameter $f_w$, velocity profiles have been drawn for various values of power law index $n$.

**CONCLUSIONS:**

(1) Although the method employed gives the approximate values of $f'_1(\eta)$, the nature of the velocity profiles can be well judged.

(2) The effect of suction is to decrease the velocity and increase the skin friction, while reverse nature occurs with injection.

(3) For fixed value of suction /injection parameter $f_w$ as power law index increases then skin friction $c_f^*$ decreases as $n$ increases.

(4) For fixed value of suction /injection parameter $f_w$ as power law index increases then velocity increases as $n$ increases.

**REFERENCES:**


