SOLVING FULLY FUZZY LINEAR PROGRAMMING PROBLEM USING TRAPEZOIDAL RANKING FUNCTION

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Abstract: In this paper fully fuzzy linear programming problem for Trapezoidal fuzzy Number with the help of linear system of equation using the linear ranking function is discussed. Here we propose a new method to find the fuzzy optimal solution of fully fuzzy linear programming problem. This is illustrated in this paper with a typical numerical example.

Key words: Fully fuzzy linear programming problem, ranking functions, trapezoidal fuzzy numbers.

1. INTRODUCTION

Linear programming is an important tool used by decision makers. This is applied frequently in applied operational research model in real world problems. Fuzzy set theory has been applied in many disciplines such as control theory, management science, mathematical modelling and industrial application. The concept of fuzzy linear programming (FLP) was first proposed by Tanaka et.al.,[8]. The first formulation of fuzzy linear programming (FLP) was proposed by Zimmerman[13]. The fuzzy optimal solution of fuzzy programming problem was proposed by Barkha Sharma and Rajendra Dubey[2]. The bounded linear programs with trapezoidal fuzzy numbers was proposed by Ebrahimnejad et.al.[4]. Ranking function methods for solving fuzzy Linear programming problems was proposed by Iden Hassan alkanani and Farrah alaa adnan[5]. Ranking function and their applications to (FLP) was proposed by Maleki.H.R[6]. The Linear programming with fuzzy variables, fuzzy sets and systems was proposed by Maleki.H.R,Tata.M and Mashinchi.M[7]. An arithmetic operations on Trapezoidal fuzzy numbers was proposed by Vahidi.J,Rezvani.S[9]. The fuzzy set theory and its applications was proposed by Zimmermann.H.J[10]. The fuzzy programming with several objective functions, fuzzy sets and systems was proposed by Zimmermann.H.J[11]. Later, many authors considered various types of FLP problems and proposed several approaches for solving these problems. As in [3], by using a general linear ranking function, we consider a fuzzy linear programming problem with Trapezoidal fuzzy numbers and solve by matrices with ranking and crisp method. Our main contribution here is the establishment of a new method for solving the FFLP problem using ranking function. Moreover, we have illustrated our method for easy understanding.

2. PRELIMINARIES

2.1 Fuzzy Set [10]

The characteristic functions $\mu_A$ of crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in $X$. This function can be generalized to a function $\mu_A$ such that the value assigned to the all of the universal set $X$ fall within a specified range .

(i.e) $\mu_A : X \rightarrow [0,1]$ the assigned values $\mu_A(x)$ is called the membership function and the set , $A=\{(x, \mu_A(x)) : x \in X\}$ defined by $\mu_A(x)$ for $x \in X$ is called fuzzy set.

2.2 Fuzzy Matrix [1]

A matrix $\bar{A} = (\bar{a}_{ij})$ is called the fuzzy matrix. If each element of $\bar{A}$ is a fuzzy number. $\bar{A}$ will be positive (negative) and denoted by $\bar{A} > 0$ ($\bar{A} < 0$) if each element of $\bar{A}$ be positive(negative). $\bar{A}$ will be a non-negative (non-positive) and denoted by $\bar{A} \leq 0$ ($\bar{A} \geq 0$) if each element of $\bar{A}$ be non-positive (non-negative).we may represent $n \times m$ fuzzy metrics $\bar{A} = (\bar{a}_{ij})_{n \times m}$

where $\bar{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$.

2.3 Trapezoidal Number [4]

A fuzzy number $\bar{A} = (m_1, n_1, \alpha_1, \beta_1)$ is said to be Trapezoidal number if its membership function is given by,
Consider the following FFLPP and solve it by the proposed method

\[
\mu_A(x) = \begin{cases} 
\frac{x-(m_1-n_1)}{\alpha_1}, & \text{if } m_1 - a_1 \leq x \leq m_1 \\
1, & \text{if } m_1 \leq x \leq n_1 \\
\frac{(n_1+\beta_1)-x}{\beta_1}, & \text{if } n_1 \leq x \leq n_1 + \beta_1 \\
0, & \text{otherwise}
\end{cases}
\]

2.4 Ranking Function [4]

Function \( R : F(R) \rightarrow R \) which maps each fuzzy number into the real line where a natural order exist.

We define order on \( F(R) \) by:
- \( \bar{A} \geq \bar{B} \iff R(\bar{A}) \geq R(\bar{B}) \)
- \( \bar{A} \leq \bar{B} \iff R(\bar{A}) < R(\bar{B}) \)
- \( \bar{A} = \bar{B} \iff R(\bar{A}) = R(\bar{B}) \)

Where \( \bar{A} \) and \( \bar{B} \) are in \( F(R) \)

We restrict our attention to linear ranking function . (i.e) a ranking function \( R \) such that ,
- \( R(K\bar{A} + \bar{B}) = KR(\bar{A}) + R(\bar{B}) \) \( \forall \bar{A}, \bar{B} \in F(R) \).

2.5 Maleki Ranking Function [6]

Let \( \bar{a} = (a^l, a^u, \alpha, \beta) \) be a fuzzy number, then the ranking function is:
- \( R(\bar{a}) = \int_0^1 (\inf \bar{a}_\lambda + \sup \bar{a}_\lambda) d\lambda \)
- \( R(\bar{a}) = a^l + a^u + \frac{\beta - \alpha}{2} \) where \( \alpha = \beta \) or \( \alpha \neq \beta \)

Which reduces to our fuzzy number \( \bar{A} = (m_1, n_1, \alpha_1, \beta_1) \) is,
- \( R(\bar{A}) = m_1 + n_1 + \frac{1}{2} (\beta_1 - \alpha_1) \), where \( \alpha_1 = \beta_1 \) or \( \alpha_1 \neq \beta_1 \)

2.6 Arithmetic Operation On Trapezoidal Fuzzy Number [1]

Let \( \bar{A} = (m_1, n_1, \alpha_1, \beta_1) \) and \( \bar{B} = (m_2, n_2, \alpha_2, \beta_2) \) be two trapezoidal fuzzy number then
- (i) \( \bar{A} \odot \bar{B} = (m_1 + m_2, n_1 + n_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2) \)
- (ii) \( \bar{A} \ominus \bar{B} = (m_1 - m_2, n_1 - n_2, \alpha_1 - \alpha_2, \beta_1 - \beta_2) \)
- (iii) \( -\bar{A} = (-m_1, -n_1, -\alpha_1, -\beta_1) \)
- (iv) If \( \bar{A} \geq 0 \) and \( \bar{B} \geq 0 \) then
  \( \bar{A} \odot \bar{B} = (m_1 m_2, n_1 n_2, \alpha_1 \alpha_2, \beta_1 \beta_2) \)

3. PROPOSED METHOD

Generally the crisp linear system of equation \( Ax=b \) is solved as a row reduced echelon form.

Whereas in this method we solve a fully fuzzy linear system \( \bar{A} \odot \bar{x} = \bar{b} \).

Assuming \( \bar{A} = (m_1, n_1, \alpha_1, \beta_1) \geq 0 \), \( x = (x_1, x_2, x_3, x_4) \geq 0 \) and \( \bar{B} = (m_2, n_2, \alpha_2, \beta_2) \geq 0 \), which can be written as
- \( (m_1, n_1, \alpha_1, \beta_1) \odot (x_1, x_2, x_3, x_4) = (m_2, n_2, \alpha_2, \beta_2) \)
- \( \Rightarrow m_1 x = m_2 \); \( n_1 y = n_2 \); \( \alpha_1 z = \alpha_2 \); \( \beta_1 w = \beta_2 \).

The following are the steps to find the solution of FFLP:

Step 1: Consider a fully fuzzy linear programming problem \( \bar{A} \odot \bar{x} = \bar{b} \).

Step 2: Take the augmented matrices \( (m_1, m_2) \), \( (n_1, n_2) \), \( (\alpha_1, \alpha_2) \) and \( (\beta_1, \beta_2) \).

Step 3: Then find \( \bar{x} = (x_1, y_1, z_1, w_1) \) for all \( i = 1 \& 2 \) by using row reduced echelon form.

Step 4: Substituting the \( \bar{x} \) values in the objective function then we obtain the solution of \( Z \).

Step 5: Take \( \bar{A} = (m_1, n_1, \alpha_1, \beta_1) \) and \( \bar{B} = (m_2, n_2, \alpha_2, \beta_2) \) as Trapezoidal fuzzy numbers.

Step 6: Then the order on \( F(R) \) is checked with the help of the Maleki ranking function.

Step 7: The order on \( F(R) \) is calculated as \( R(\bar{A}) \) and \( R(\bar{B}) \) by the ranking function
- \( R(\bar{A}) = m_1 + n_1 + \frac{1}{2} (\beta_1 - \alpha_1) \) and
- \( R(\bar{B}) = m_2 + n_2 + \frac{1}{2} (\beta_2 - \alpha_2) \).

Step 8: Then the order on \( F(R) \) is checked with the linear ranking function.

4. NUMERICAL EXAMPLE

Consider the following FFLPP and solve it by the proposed method
Max \( Z = [2,3,1,7] \bar{x}_1 + [4,5,1,3] \bar{x}_2 \)

subject to

\[
(3,6,4,2) \otimes x_1 \oplus (4,6,3,1) \otimes x_2 \leq (27,66,26,15) \\
(4,5,3,1) \otimes x_1 \oplus (5,8,3,1) \otimes x_2 \leq (35,70,24,12) \\
x_1, x_2 \geq 0
\]

**Solution:**

The given FFLPP can be written as

\[
\begin{bmatrix}
(3,6,4,2) & (4,6,3,1) \\
(4,5,3,1) & (5,8,3,1)
\end{bmatrix} \begin{bmatrix}
\bar{x}_1 \\
\bar{x}_2
\end{bmatrix} = \begin{bmatrix}
(27,66,26,15) \\
(35,70,24,12)
\end{bmatrix}
\]

Where, \( m_1 = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \); \( n_1 = \begin{bmatrix} 6 & 6 \\ 5 & 8 \end{bmatrix} \); \( \alpha_1 = \begin{bmatrix} 4 & 3 \\ 3 & 1 \end{bmatrix} \); \( \beta_1 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \); \( m_2 = \begin{bmatrix} 27 \\ 35 \end{bmatrix} \); \( n_2 = \begin{bmatrix} 66 \\ 70 \end{bmatrix} \);

\( \alpha_2 = \begin{bmatrix} 26 \\ 24 \end{bmatrix} \); \( \beta_2 = \begin{bmatrix} 15 \\ 12 \end{bmatrix} \);

The augmented matrix \((m_1, m_2)\) is

\[
\begin{bmatrix}
3 & 4 & 27 \\
4 & 5 & 35
\end{bmatrix}
\]

The row reduced echelon form of this matrix is obtained as follows:

\[
\begin{bmatrix}
3 & 4 & 27 \\
4 & 5 & 35
\end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 11 \\
0 & 0 & 1 \end{bmatrix}
\]

\[
R_2 \rightarrow R_2 - R_1
\]

\[
\begin{bmatrix}
1 & 1 & 11 \\
0 & 1 & 3
\end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 5 \\
0 & 1 & 3
\end{bmatrix}
\]

\[
R_1 \rightarrow -R_1
\]

Using the row reduced echelon form of the augmented matrix \((m_1, m_2)\), the obtained value is \( x_1 = 5; x_2 = 3 \).

Similarly the row reduced echelon form of the augmented matrix \((n_1, n_2)\) is

\[
\begin{bmatrix}
6 & 6 & 66 \\
5 & 8 & 70
\end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 11 \\
0 & 0 & 15 \end{bmatrix}
\]

\[
R_2 \rightarrow R_2 - 5R_1
\]

\[
\begin{bmatrix} 1 & 1 & 11 \\
0 & 0 & 15 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 6 \\
0 & 0 & 5 \end{bmatrix}
\]

\[
R_1 \rightarrow R_1 - R_2
\]

Using the row reduced echelon form of the augmented matrix \((n_1, n_2)\), the obtained value is \( y_1 = 6; y_2 = 5 \).

Similarly the row reduced echelon form of the augmented matrix \((\alpha_1, \alpha_2)\) is,

\[
\begin{bmatrix}
4 & 3 & 26 \\
3 & 3 & 24
\end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 3/4 & 13/2 \\
0 & 3/4 & 9/2 \end{bmatrix}
\]

\[
R_1 \rightarrow R_1/4
\]

\[
\begin{bmatrix} 1 & 3/4 & 13/2 \\
0 & 3/4 & 9/2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\
0 & 1 & 6 \end{bmatrix}
\]

\[
R_2 \rightarrow 4/3 R_2
\]

\( Z_1 = 2; Z_2 = 6 \).

Finally, the row reduced echelon form of the augmented matrix \((\beta_1, \beta_2)\) is,
\[
\begin{bmatrix}
2 & 1 & 15 \\
1 & 1 & 12
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
1 & 1/2 & 15/2 \\
1 & 1 & 12
\end{bmatrix}
R_1 \rightarrow R_1^{1/2}
\]
\[
\begin{bmatrix}
0 & 1/2 & 9/2 \\
1 & 1 & 3
\end{bmatrix}
R_2 \rightarrow R_2 - R_1
\]
\[
\begin{bmatrix}
0 & 1/2 & 9/2 \\
0 & 1 & 3
\end{bmatrix}
R_1 \rightarrow R_1 - R_2
\]
\[
\begin{bmatrix}
0 & 1/2 & 9/2 \\
0 & 1 & 3
\end{bmatrix}
R_2 \rightarrow 2R_2
\]

\[w_1 = 3; \ w_2 = 9\]

Substituting appropriate value in \(x_i = (x_i, y_i, z_i, w_i)\) for all \(i = 1 \& 2\) \(x_1 = (5, 6, 2, 3)\) and \(x_2 = (3, 5, 6, 9)\)

Max \(Z = [2, 3, 1, 7] \otimes [5, 6, 2, 3] + [4, 5, 1, 3] \otimes [3, 5, 6, 9]\)

\[(10+12),(18+25),(2+6),(21+27)\]

\[= (22, 43, 8, 48)\]

By using Maleki ranking function, we have

\[R(22, 43, 8, 48) = (22+43)+\frac{1}{2}(48-8)\]

\[= 65 + \frac{1}{2}(40)\]

\[= 65 + 20\]

\[= 85\]

Now \(R(2, 3, 1, 7) = (2+3)+\frac{1}{2}(7-1)\)

\[= 5 + \frac{1}{2}(6)\]

\[= 5 + 3\]

\[= 8\]

\[R(4, 5, 1, 3) = (4+5)+\frac{1}{2}(3-1)\]

\[= 9 + \frac{1}{2}(2)\]

\[= 9 + 1\]

\[= 10\]

Now the problem becomes the crisp LPP as

Max \(Z = 8x_1 + 10x_2\)

subject to

\[8x_1 + 9x_2 = 87.5\]

\[8x_1 + 12x_2 = 99\]

\[x_1, x_2 \geq 0\]

The solution is

Max \(Z = 91.33, \ x_1 = 6.63, \ x_2 = 3.83\)

Now the order on \(F(R)\), where \(\tilde{A}\) and \(\tilde{B}\) are in \(F(R)\) is verified,

(i) \(\tilde{A} \geq \tilde{B} \iff R(\tilde{A}) \geq R(\tilde{B})\)

Suppose \(\tilde{A} = (4, 6, 3, 1)\) and \(\tilde{B} = (3, 6, 4, 2)\)

Now, \(\tilde{A} \geq \tilde{B} \iff R(4, 6, 3, 1) \geq R(3, 6, 4, 2)\).

By using Maleki ranking function, we have

\[R(\tilde{A}) = m_1 + n_1 + \frac{1}{2} (\beta_1 - \alpha_1)\]

\[= 4 + 6 + \frac{1}{2}(1-3)\]

\[= 10 + \frac{1}{2}(2)\]

\[= 9\]

Similarly, \(R(\tilde{B}) = m_2 + n_2 + \frac{1}{2} (\beta_2 - \alpha_2)\)

\[= 3 + 6 + \frac{1}{2}(2-4)\]

\[= 9 + \frac{1}{2}(2)\]

\[= 8\]

Which gives, \(R(\tilde{A}) \geq R(\tilde{B})\)
Hence, $A \geq B \iff R(\bar{A}) \geq R(\bar{B})$

(ii) $A \leq B \iff R(\bar{A}) < R(\bar{B})$  
Assume, $\bar{A} = (4,5,3,1)$ and $\bar{B} = (5,8,3,1)$  
$R(\bar{A}) = 4+5+\frac{1}{2}(1-3)$  
\hspace{2cm} = 9+\frac{1}{2}(-2)  
\hspace{2cm} = 9 - 1  
\hspace{2cm} = 8$

Similarly, $R(\bar{B}) = 5+8+\frac{1}{2}(1-3)$  
\hspace{2cm} = 13+\frac{1}{2}(-2)  
\hspace{2cm} = 13 - 1  
\hspace{2cm} = 12$

Hence, $A \leq B \iff R(\bar{A}) < R(\bar{B})$

(iii) $A \equiv B \iff R(\bar{A}) = R(\bar{B})$  
Suppose, $\bar{A} = (4,6,3,1)$ and $\bar{B} = (4,6,3,1)$  
$R(\bar{A}) = 4+6+\frac{1}{2}(1-3)$  
\hspace{2cm} = 10+\frac{1}{2}(-2)  
\hspace{2cm} = 10 - 1  
\hspace{2cm} = 9$

Similarly, $R(\bar{B}) = 4+6+\frac{1}{2}(1-3)$  
\hspace{2cm} = 10+\frac{1}{2}(-2)  
\hspace{2cm} = 10 - 1  
\hspace{2cm} = 9$

Therefore $R(\bar{A}) = R(\bar{B})$  
Hence $A \equiv B \iff R(\bar{A}) = R(\bar{B})$.

(iv) $R(KA + B) = KR(\bar{A}) + R(\bar{B}) \land \bar{A}, \bar{B} \in F(R)$

Suppose, $\bar{A} = (4,6,3,1)$ and $\bar{B} = (3,6,4,2)$

Now taking L.H.S: For $(k=2)$, we have,

$R(KA + B) = R(2(4,6,3,1)+(3,6,4,2))$  
\hspace{2cm} = R ((8,12,6,2)+(3,6,4,2))  
\hspace{2cm} = R (11,18,10,4)$  
\hspace{2cm} = 11+18+\frac{1}{2}(4-10)$  
\hspace{2cm} =29+\frac{1}{2}(-6)  
\hspace{2cm} =29.3  
\hspace{2cm} =26$

Taking R.H.S:  
For $k=2$, we have,

$KR(\bar{A}) + R(\bar{B}) = 2R((4,6,3,1)+R(3,6,4,2))$  
\hspace{2cm} =2[4+6+\frac{1}{2}(1-3)]+R(3,6,4,2)$  
\hspace{2cm} =2[(4+6+1)]+(3+6+\frac{1}{2}(2-4)]  
\hspace{2cm} =2[9]+8  
\hspace{2cm} =18+8  
\hspace{2cm} =26$

Therefore L.H.S = R.H.S.
\[ (i.e) \ R(KA + B) = KR(A) + R(B) \ \forall \ A, B \in F(R). \]

CONCLUSION

In this paper linear ranking function method is proposed to find the fuzzy optimal solution of Fully fuzzy linear programming problem (FFLP) with equality constraints by representing all the parameter as trapezoidal fuzzy numbers. Here row reduced echelon form of matrices was used to construct a new method for solving FFLP problem and using the general linear ranking functions on fuzzy numbers. The proposed method is easy to understand and can be applied in real life situations. This can be further extended in real life problems along with hexagonal and octagonal fuzzy numbers.

REFERENCES


