RANKING OF TRIANGULAR FUZZY NUMBER METHOD TO SOLVE AN UNBALANCED ASSIGNMENT PROBLEM

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Abstract: In this paper, we proposed to solve the fuzzy salesman problem using Hungarian method. An unbalanced assignment problem is solved using Ranking of Triangular Fuzzy number. In this paper, first the proposed unbalanced assignment problem is formulated to a crisp assignment problem and solved by using Hungarian method and using Ranking of Triangular Fuzzy number. Numerical examples show that the fuzzy ranking method offers an effective tool for handling the unbalanced assignment problem.

Keywords: Fuzzy sets, Fuzzy unbalanced Assignment problem, Ranking of Triangular Fuzzy number, membership function.

1. INTRODUCTION
The Fuzzy Assignment problem is a special type of fuzzy linear programming problem and it is a subclass of fuzzy transportation problem. The Fuzzy Assignment problem can be stated as follows:

Let n number of jobs is performed by number of persons, where the costs depend on the specific assignments. Each job must be assigned to one and only one worker and each worker has to perform one and only one job.

The problem is to find such an assignment so that the total cost is optimized. The fuzzy assignment problem can be applied to nxn fuzzy cost matrix (C_{ij}), where C_{ij} represents the fuzzy cost associated with worker (i=1,2,3,…n) who has performed job (j=1,2,3,…n). The fuzzy assignment problem when costs are fuzzy numbers can also be modelled as 0-1 integer programming problem. The fuzzy unbalanced assignment problems can be solved by the method proposed for unbalanced assignment method.

The unbalanced assignment problem can be changed to balanced assignment problem and after solving the problem by assignment technique we use the method of triangular fuzzy number method.

In this paper, the procedure is best illustrated with the help of a sample problem.

2. Preliminaries
3. In this section some basic definitions and arithmetic operations are reviewed.

Assignment Problem:
Assignment problem is a special case of the transportation problem in which the number of square and destination are the same and the objective is to assign the given job to most appropriate person so as to optimize the objective function like minimize cost.

Balanced Assignment Problem:
When the number of rows equals to the number of columns.
Number of rows = Number of columns.

Unbalanced Assignment Problem:
When the number of rows not equals to the number of columns and vice versa.

Number of rows ≠ Number of columns.

Dummy Matrix:
We introduce dummy rows / columns in the matrix. These rows or columns have a zero cost elements. Thus we can balance the problem.

Row Reduction:
Row reduction subtracts the minimum cost of each row of the cost matrix from all the element of the respective row of the resulting matrix.
Column Reduction:
Column reduction subtracts the minimum cost of each column of the cost matrix from all the element of the respective column of the resulting matrix.

Fuzzy set:
A fuzzy set A on a set x is characterised by a mapping m: x → [0, 1] called the membership function. A fuzzy set is denoted as A=(x, m).

Membership Function:
Let x denotes the universal set of the membership function.
\[\mu_A = \begin{cases} 
1, & x \in A \\
0, & x \notin A 
\end{cases}\]

Triangular Fuzzy number:
A fuzzy number A=(a,b,c) is said to be Triangular fuzzy number of its membership function given by
\[\mu_A(x) = \begin{cases} 
\frac{x-a}{b-a}, & a \leq x \leq b \\
\frac{c-x}{c-b}, & b \leq x \leq c \\
0, & \text{otherwise}
\end{cases}\]

Ranking of triangular fuzzy number:
Several approaches for the ranking of fuzzy numbers have been proposed in the literature. An efficient approach for comparing the fuzzy number is by the use of a ranking function based on their graded means.

That is for every A=(a(1), a(2), a(3)) ∈ F(R) the ranking function R=F(R)→R by graded means is defined a function.
\[R(A) = \frac{a_1 + a_2 + a_3}{3}\]

Robust’s Ranking Method:
Robust’s ranking technique which satisfies compensation given a convex fuzzy number, the Robust’s ranking index is defined as
\[R(C^a) = \int_0^a (C_a^l, C_a^u) d\alpha\] where \((C_a^l, C_a^u)\) is the \(a\)-level cut of the fuzzy number.

\(\alpha\)-cut of a fuzzy number:
The \(\alpha\)-cut of a fuzzy number A(x) is defined as A(\(\alpha\)) = \{ x : \mu(x) \geq \alpha; \ \alpha \in [0,1] \}

Mathematical formulation of Fuzzy Assignment Problem:
Mathematically, the fuzzy assignment problem is,
\[\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} x_{ij}\]
Subject to the constraints;
\[\sum_{j=1}^{m} x_{ij} = 1; \ i=1,2,\ldots,m\]
\[\sum_{i=1}^{n} x_{ij} = 1; \ j=1,2,\ldots,n\]
\[x_{ij} = \begin{cases} 
1, & \text{if } i^{th} \text{ person is assigned } j^{th} \text{ work} \\
0, & \text{otherwise}
\end{cases}\]
where \(x_{ij}\) denotes that \(j^{th}\) work is to be assigned to the \(i^{th}\) person.

Mathematical formulation of the Assignment Problem:
Consider a problem of assignment of n resources (persons) to n-activities (jobs) so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job. The cost matrix \((C_{ij})\) is given as follows

<table>
<thead>
<tr>
<th>Resource</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>.</th>
<th>.</th>
<th>An</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>C_{11}</td>
<td>C_{12}</td>
<td>C_{13}</td>
<td>.</td>
<td>.</td>
<td>C_{1n}</td>
</tr>
<tr>
<td>R2</td>
<td>C_{21}</td>
<td>C_{22}</td>
<td>C_{23}</td>
<td>.</td>
<td>.</td>
<td>C_{2n}</td>
</tr>
<tr>
<td>R3</td>
<td>C_{31}</td>
<td>C_{32}</td>
<td>C_{33}</td>
<td>.</td>
<td>.</td>
<td>C_{3n}</td>
</tr>
</tbody>
</table>
This cost matrix is same as that of a transportation problem except that availability at each of the resources and the requirement at each of the destination is unity (due to the fact that assignments are made on a one-to-one basis).

Let \( x_{ij} \) denotes the assignment of \( i^{th} \) resource to \( j^{th} \) activity, such that

\[
X_{ij} = \begin{cases} 
1, \text{if resource } i \text{ is assigned to activity } j \\
0, \text{otherwise}
\end{cases}
\]

Then the Mathematical formulation of the Assignment Problem is

\[
\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}
\]

Subject to the constraints:

\[
\sum_{i=1}^{m} x_{ij} = 1; \quad i = 1, 2, \ldots, m
\]

\[
\sum_{j=1}^{n} x_{ij} = 1; \quad j = 1, 2, \ldots, n
\]

4. UNBALANCED ASSIGNMENT PROBLEM TO CHANGE INTO BALANCED ASSIGNMENT PROBLEM

The number of rows is not equal to the number of columns, then the problem is termed as unbalanced assignment problem then this problem changed into balanced assignment problem as follows necessary number of dummy rows / columns are added such that the cost matrix is a square matrix, the values for the entries in the dummy rows / columns are assumed to be zero.

5. NUMERICAL EXAMPLE

Let us consider a fuzzy unbalanced assignment problem with rows representing 4 area \( A_1, A_2, A_3, A_4 \) and columns representing the salesman’s \( B_1, B_2, B_3 \). The cost matrix \( (\tilde{C}_{ij}) \) is given whose elements are triangular fuzzy numbers. The problem is to find the optimal assignment so that the total cost of area assignment becomes minimum.

\[
(\tilde{C}_{ij}) = \begin{pmatrix}
A_1 & B_1 & B_2 & B_3 \\
-2,1,4 & 1,4,7 & 6,7,8 \\
6,10,14 & 2,5,8 & 7,8,9 \\
6,7,8 & 10,12,14 & 5,6,7 \\
-2,3,8 & 7,8,9 & 8,9,10
\end{pmatrix}
\]

Solution:
The given problem is a fuzzy unbalanced assignment problem. We have to change into the fuzzy balanced assignment problem as follows.

\[
(\tilde{C}_{ij}) = \begin{pmatrix}
A_1 & B_1 & B_2 & B_3 & B_4 \\
-2,1,4 & 1,4,7 & 6,7,8 & 0,0,0 \\
6,10,14 & 2,5,8 & 7,8,9 & 0,0,0 \\
6,7,8 & 10,12,14 & 5,6,7 & 0,0,0 \\
-2,3,8 & 7,8,9 & 8,9,10 & 0,0,0
\end{pmatrix}
\]

The fuzzy balanced assignment problem can be formulated in the following mathematical programming form.

\[
\text{Min} \{ R(-2,1,4)x_{11} + R(1,4,7)x_{12} + R(6,7,8)x_{13} + R(0,0,0)x_{14} + R(6,10,14)x_{21} + R(2,5,8)x_{22} + R(7,8,9)x_{23} + R(0,0,0)x_{24} + R(6,7,8)x_{31} + R(10,12,14)x_{32} + R(5,6,7)x_{33} + R(0,0,0)x_{34} + R(-2,3,8)x_{41} + R(7,8,9)x_{42} + R(8,9,10)x_{43} + R(0,0,0)x_{44} \}
\]

Subject to the constraints:
\[ x_{11} + x_{12} + x_{13} + x_{14} = 1 \\
\[ x_{11} + x_{21} + x_{31} + x_{41} = 1 \\
\[ x_{21} + x_{22} + x_{32} + x_{42} = 1 \\
\[ x_{21} + x_{22} + x_{32} + x_{42} = 1 \\
\[ x_{31} + x_{32} + x_{33} + x_{34} = 1 \\
\[ x_{31} + x_{32} + x_{33} + x_{34} = 1 \\
\[ x_{41} + x_{42} + x_{43} + x_{44} = 1 \\
\[ x_{41} + x_{42} + x_{43} + x_{44} = 1 \\
\[ x_{ij} \in [0,1] 
\]

Now, we conclude \( R(-2,1,4) \) by applying Robust’s Ranking method. The membership function of the triangular fuzzy number \((-2,1,4)\) is

\[
\mu(x) = \begin{cases} 
\frac{x+2}{1+2} &: -2 \leq x \leq 1 \\
\frac{4-x}{4-1} &: 1 \leq x \leq 4 \\
0 &: \text{otherwise}
\end{cases}
\]

\[
\mu(x) = \begin{cases} 
\frac{x+2}{3} &: -2 \leq x \leq 1 \\
\frac{4-x}{3} &: 1 \leq x \leq 4 \\
0 &: \text{otherwise}
\end{cases}
\]

Ranking of Triangular fuzzy number

\[ \mathcal{R}(A) = \frac{a_1 + a_2 + a_3}{6} \]

\[ \mathcal{R}(C_{11}) = \frac{-2+4(1)+4}{6} = \frac{-2+4}{6} = \frac{6}{6} = 1 \]

Proceeding similarly, the ranking of triangular fuzzy number for the fuzzy costs \((C_{ij})\) are calculated as follows:

\[ \mathcal{R}(C_{12}) = \frac{1+4(4)+7}{6} = \frac{1+16+7}{6} = \frac{24}{6} = 4 \]

\[ \mathcal{R}(C_{13}) = \frac{6+4(7)+8}{6} = \frac{6+28+8}{6} = \frac{42}{6} = 7 \]

\[ \mathcal{R}(C_{14}) = \frac{0+4(0)+0}{6} = 0 \]

\[ \mathcal{R}(C_{21}) = \frac{6+4(10)+14}{6} = \frac{6+40+14}{6} = \frac{60}{6} = 10 \]

\[ \mathcal{R}(C_{22}) = \frac{2+4(5)+8}{6} = \frac{2+20+8}{6} = \frac{30}{6} = 5 \]

\[ \mathcal{R}(C_{23}) = \frac{7+4(8)+8}{6} = \frac{7+32+9}{6} = \frac{48}{6} = 8 \]

\[ \mathcal{R}(C_{24}) = \frac{0+4(0)+0}{6} = 0 \]
We replace these values for the corresponding \( \overline{C_{ij}} \) which results in assignment problem in the linear programming problem.

We solve it by Hungarian method to get the following optimal solution.

\[
(\overline{C_{ij}}) = \begin{pmatrix}
1 & 4 & 7 & 0 \\
10 & 5 & 8 & 0 \\
7 & 12 & 6 & 0 \\
3 & 8 & 9 & 0 \\
\end{pmatrix}
\]

Row Reduction:

\[
(\overline{C_{ij}}) = \begin{pmatrix}
1 & 4 & 7 & 0 \\
10 & 5 & 8 & 0 \\
7 & 12 & 6 & 0 \\
3 & 8 & 9 & 0 \\
\end{pmatrix}
\]

Column Reduction:

\[
(\overline{C_{ij}}) = \begin{pmatrix}
0 & 0 & 1 & 0 \\
9 & 1 & 2 & 0 \\
6 & 8 & 0 & 0 \\
2 & 4 & 3 & 0 \\
\end{pmatrix}
\]

By using Hungarian Assignment Method:

\[
(\overline{C_{ij}}) = \begin{pmatrix}
0 & 0 & 1 & 1 \\
8 & 0 & 1 & 0 \\
6 & 8 & 0 & 1 \\
1 & 3 & 2 & 0 \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
0 & 1 & 1 \\
8 & 0 & 1 \\
6 & 8 & 0 \\
1 & 3 & 2 \\
\end{pmatrix}
\]

The optimal Assignment

\[A_1 \rightarrow B_1, \ A_2 \rightarrow B_2, \ A_3 \rightarrow B_3, \ A_4 \rightarrow B_4\]

The optimal total minimum cost = Rs. 1+5+6+0 = Rs. 12

The fuzzy optimal Assignment

\[A_1 \rightarrow B_1, \ A_2 \rightarrow B_2, \ A_3 \rightarrow B_3, \ A_4 \rightarrow B_4\]

The fuzzy optimal total minimum cost = \(\widetilde{\mathbf{C}}_{11} + \widetilde{\mathbf{C}}_{22} + \widetilde{\mathbf{C}}_{33} + \widetilde{\mathbf{C}}_{44}\)

\[= R(5,12,19)\]

\[= \text{Rs. 12}\]

CONCLUSION

In this paper, the fuzzy unbalanced assignment problem has been transformed into crisp assignment problem using Ranking of triangular fuzzy number. Numerical example shows that by using this method we can have the optimal assignment as well as the fuzzy optimal total cost. By using Ranking of Triangular fuzzy number methods, we have shown that the total cost obtained is optimal moreover; one can conclude that the solution of fuzzy problems can be obtained by triangular fuzzy number method effectively.

REFERENCE