Existence and Stability of Fixed Points of Resource Dynamics Subjected to Additive Allee effect

Birku Demewoz Misganaw
Department of Mathematics, Gondar University, Gondar, Ethiopia.

ABSTRACT. Allee effect refers to a reduction in individual fitness at low population density that can lead to extinction. These effects occur naturally in many real world applications. In this article we are interested in studying stability behaviour of the equilibrium solutions of the resource dynamics subjected to additive Allee effects with out and with harvesting.

KEY WORDS: Renewable resource, Allee effect, additive Allee effect, equilibrium solution.

1. Introduction

Allee effects generally refer to a reduction in individual fitness at low population size or density [3]. Populations can show Allee dynamics due to a wide range of biological phenomenon such as reduced anti predator vigilance, mating difficulty, reduced defence anti predator, deficient feeding to low densities, and so on [4], genetic drift, loss of genetic variability, demographic fluctuations stochasticity [3, 5]. There are several mechanisms which generate Allee effects and a classification of these effects are presented in [1]. If the growth of a resource is represented by a depensation (or critical depensation) curve it is said to exhibit Allee effects [2]. These Allee effects may lead to threshold population densities, below which the population growth is negative and the population is likely to go extinct.
2. Resource Dynamics

Let us consider a model with the growth equation describes dynamics of renewable resources such as a resource with additive Allee effect given by:

\[ \frac{dy}{dt} = y \left( 1 - y - \frac{\eta}{1 + my} \right) \]  

(2.1)

where \( m \) is a cooperative parameter and \( \eta \) is extra mortality rate. Eq. (2.1) also referred to us an equation exhibiting additive Allee effect.

**Lemma 1**: The existence and stability of equilibrium solutions for Eq. (2.1) depends on \( m \) and \( \eta \).

**Proof.** The non trivial equilibrium points in terms of \( m \) and \( \eta \) are.

\[ y = \frac{m - 1 \pm \sqrt{(m - 1)^2 + 4m(1 - \eta)}}{2m} \]

Consider

\[ (m - 1)^2 + 4m(1 - \eta) = 0 \]

\[ \Rightarrow \eta = \frac{(m - 1)^2}{4m} + 1 \]

The stability and instability of equilibrium solutions are verified in the following figure.\[ \square \]
Figure 1: Figure represents discriminant of equilibrium solutions \( \eta = \frac{(m-1)^2}{4m} \) + 1 and the resource dynamic equation \( \frac{dy}{dt} = y \left( 1 - y - \frac{\eta}{1+my} \right) \).
3. Additive Allee effect subject to Harvesting

Let us assume that the resource is subject to proportionate harvesting, i.e., the catch is proportional to the stock and effort. Under this assumption equation (1) gets modified to:

\[
\frac{dy}{dt} = y \left(1 - y - \frac{\eta}{1 + my}\right) - q\bar{E}y
\]  

\[\text{(3.1)}\]

where \(\bar{E}\) represents effort and \(q\) stands for catchability coefficient. The above model can be written as:

\[
\frac{dy}{dt} = y \left(1 - y - \frac{\eta}{1 + my}\right) - Ey
\]

\[\text{(3.2)}\]

where \(E = q\bar{E}\)

**Lemma 2**: The existence and stability of equilibrium solutions for Eq. (3.2) depends on \(m\), \(\eta\) and \(E\).

*Proof.* The non trivial equilibrium points in terms of \(m\), \(\eta\) and \(E\) are.

\[
y = \frac{m(1 - E) - 1 \pm \sqrt{(m - mE - 1)^2 + 4m(1 - E - \eta)}}{2m}
\]

Let us consider:

\[
(m - mE - 1)^2 + 4m(1 - E - \eta) = 0
\]

\[
\eta = \frac{m(1 - E)^2}{4} + 0.5 - 0.5E + \frac{1}{4m}
\]

The stability and instability of equilibrium solutions are verified by the following figure. \(\Box\)
Figure 2: Figure represents discriminant of equilibrium solutions and the resource dynamic equation.
References


