Bulk viscous cosmological models with linearly varying deceleration parameter in a scale covariant theory of gravitation

S. Surendra Singh
Department of Mathematics, Manipur University,
Imphal - 795003, Manipur, India.
email: ssuren.mu@gmail.com

Friedmann Robertson Walker universe in the presence of viscous fluid are investigated in the cosmological theory based on the scale covariant theory of gravitation by considering the deceleration to be linearly variable. Exact solutions have been obtained from which cosmological models have been derived. The universe ends with big rip. Dynamical and physical implications of the solutions have been briefly studied.

PACS numbers: 98.80.Cq, 95.35.+d

Key-words: co-efficient of viscosity, varying deceleration parameter, big rip.

1. Introduction:

In recent years, there has been considerable interest in deriving cosmological models for alternative theories of gravity. Einstein introduced the cosmological constant into his field equations in order to obtain a static model of the universe since without the cosmological term, his field equations admit only non static cosmological models for nonzero energy density. In 1917, Einstein developed his general theory of relativity in which gravitation is described in terms of geometry. In this work, we obtain cosmological models in the scale covariant theory of Canuto et al. (1977). Beesham (1986) studies the Bianchi type-I cosmological model in the scale covariant theory. The field equation in the scale covariant theory can be derived by taking the general relativistic equations (Ellis 1971), writing the tensors in co-tensor form and replacing covariant differentiation (Canuto et al 1977). The space-time underlying the scale covariant theory is an integrable Weyl manifold as tensors form the objects of interest in general relativity, co-tensors are the corresponding quantities in the present theory. The co-covariant derivative is an extension of the covariant derivative so that the resulting quantities are a co-tensor (Dirac 1973, Canuto et al. 1977). Rahman and Banerji (1985) have given a related condition in the scale covariant theory for the absence of a singularity of zero volume or infinite density for FRW universes. Venkateswarlu and Pavan Kumar (2005) have studied higher dimensional string cosmologies in scale covariant theory of gravitation. Reddy, et al (2002) studied Friedmann universe with bulk viscosity in a scale covariant theory. In order to study the evolution of universe, many authors constructed cosmological models containing viscous fluid. The presence of viscosity in the fluid introduces many interesting features in the dynamics of homogeneous cosmological models. During the past decades, (Canuto and Goldman 1983a, 1983b; Johri and Sudharsan 1988) discussed the effect of bulk viscosity on the evolution of Friedmann models in general relatively while Pimental (1994) studied the same in Brans and Dicke (1961) scalar tensor theory. Several theories are proposed by various authors as alternative theories to reveal the nature of the universe at the early stage of evolution. Ibotombi et al. (2010) also discussed a new class of bulk viscous cosmological models in a scale covariant theory of gravitation. The SNIa-type supernovae observations, the large scale structures and the cosmic microwave background (CMB) radiations confirmed that the present universe is not only expanding but also accelerating. There is a consensus on the conclusion that the universe has entered a state of accelerating expansion at redshift $z \sim 0.5$. In two recent studies by Cunha and Lima (2008) and Cunha(2009), the transition redshift to the accelerating expansion of the current universe has been examined using the kinematic approach to cosmological data analysis that provides a direct evidence to the present accelerating stage of the universe which does not depend neither on the validity of general relativity, nor on the matter-energy content of the universe. Recently Li, et al.(2011) studied the present acceleration of universe by analyzing the sample of baryonic acoustic oscillation (BAO) with (CMB) radiation and concluded that such sample of BAO with CMB increases the present cosmic acceleration which has been further explained by plotting graphs for change of deceleration parameter $q$ with redshift $z < 2$. Akarsu and Dereli (2012) proposed a linearly varying deceleration parameter (LVDP) and obtained the accelerating cosmological solutions.
by considering the spatially homogeneous and isotropic Robertson-Walker (RW) space-time filled with perfect fluid in general relativity. As a special case, LVDP also covers the special law of variation for Hubble parameter, which yields constant deceleration parameter (CDP) models of the universe presented by Berman (1983,1988).

Motivated from the studies outlined above, the purpose of the present is to study the bulk viscous universe with the help of linearly varying deceleration parameter in a scale covariant theory of gravitation. We study the field equations in Sect. 2, solutions of field equation in Sect. 3 and Physical behaviors of the model and discussion in Sect. 4. We conclude in Sect. 5.

2. The field equations:

We assume a homogeneous and isotropic cosmological model. It is natural to consider the metric tensor of this gravitational field to be of the type

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]$$  \hspace{1cm} (1)

where $a(t)$ the scale factor (the speed of light $c = 1$ and signature $+ - - -$ are used). A scale covariant theory of gravitation which admits a variable $G$ is formulated by Canuto et al. (1977) and which is a viable alternative in general relativity (Wesson 1980, Will 1984). We consider a conformal transformation:

$$g_{ij} = \phi^2(x^c) g_{ij}, \quad \phi > 0$$  \hspace{1cm} (2)

relating the metric $\bar{g}_{ij}$ determining macroscopic metric properties and $g_{ij}$ determining microscopic metric properties where $i = 1, 2, 3, 4$. We use the convention that bars denote gravitational units and unbarred atomic units. The gauge function $\phi$, in its most general formulation is a function of all space time co-ordinates. The possibilities that have been considered for gauge function (Canuto et al. 1977) are

$$\phi(t) = \left( \frac{t_0}{t} \right) \epsilon, \quad \epsilon = \pm 1, \pm \frac{1}{2}$$  \hspace{1cm} (3)

where $t_0$ is a constant. The field equations in scale covariant theory are given by

$$R_{ij} - \frac{1}{2} R g_{ij} + f_{ij}(\phi) = G T_{ij},$$  \hspace{1cm} (4)

where $f_{ij}(\phi)$ is given by

$$\phi^2 f_{ij} = -2\phi \delta_{ij} + 4\phi \epsilon_{ij} + g_{ij} (2\phi \phi^k_k - \phi^k_k)$$  \hspace{1cm} (5)

Here, $T_{ij}$ is the matter energy-momentum tensor, $g_{ij}$ the metric tensor, $R$ the scalar curvature, $R_{ij}$ the Ricci tensor and a semicolon denotes covariant differentiation. We have the energy-momentum tensor in the presence of bulk stress has the form

$$T_{ij} = -\bar{\rho} g_{ij} + (\bar{p} + \rho) u_i u_j$$  \hspace{1cm} (6)

together with co-moving coordinates $u_i u^i = 1$ and $u_i = (0, 0, 0, 1)$ and

$$\bar{p} = p - \eta u^i_i$$  \hspace{1cm} (7)

Here $\rho, p, \eta$ and $u$ are respectively the energy density, isotropic pressure, coefficient of bulk viscosity and four velocity vector of the matter distribution. In general $\eta$ is a function of time. The field equation (2) with (1), (4), (5),(6) and (7) take the form

$$3H^2 + \frac{\kappa}{a^2} - 6\left( \frac{\dot{\phi}}{\phi a} \right) - 3 \left( \frac{\dot{\phi}}{\phi} \right)^2 = G\rho$$  \hspace{1cm} (8)
\[ 2\dot{H} + 3H^2 + \frac{\kappa}{a^2} - 4\left(\frac{\dot{\phi}}{\dot{a}}\right) + \left(\frac{\dot{\phi}}{\phi}\right)^2 - \frac{2\dot{\phi}}{\phi} = -G(p - 3\eta H) \]  

(9)

where \( H = \frac{\dot{a}}{a} \) is the Hubble’s parameter. The usual energy momentum conservation relation leads to

\[ \dot{\rho} + 3(p + \rho - 3\eta H)H = 0 \]

(10)

where a dot indicates differentiation w.r.t. ‘t’. For the specification of \( \eta \), we assume that the fluid obeys a barotropic equation of state

\[ p = \gamma \rho \]

(11)

where \( \gamma(-1 \leq \gamma \leq 1) \) is a constant.

### 3. Models and solutions of the field equations

The function \( a(t) \) remains undetermined. To obtain its explicit dependence on \( t \), one may have to introduce additional assumptions. We solve the field equations (8)-(10) by assuming a linearly varying deceleration parameter as

\[ q = -\frac{\ddot{a}}{a^2} = -kt + m - 1 \]

(12)

where \( k \geq 0 \) and \( m \geq 0 \). This law has been recently proposed by Akarsu and Dereli (2012). When \( k = 0 \), eq. (12) reduces to the Berman’s law (1983, 1988). The universe will exhibit decelerating expansion if \( q > 0 \), an expansion with constant rate if \( q = 0 \), accelerating power law expansion if \(-1 < q < 0 \), exponential expansion (also known as de Sitter expansion) if \( q = -1 \) and super-exponential expansion if \( q < -1 \) (Carroll et al. 2003, Caldwell et al. 2003). Solving eq. (12) one obtains three different form of solutions for the scale factor

\[ a = \begin{cases} 
  a_1 e^{\frac{2}{\sqrt{m^2-2c_1 k}}} \arctanh \left( \frac{k t - m}{\sqrt{m^2-2c_1 k}} \right), & \text{for } k > 0 \text{ and } m \geq 0, \\
  a_2 (mt + c_2)^\frac{k}{m}, & \text{for } k = 0 \text{ and } m > 0, \\
  a_3 e^{c_3 t}, & \text{for } k = 0 \text{ and } m = 0. 
\end{cases} \]

(13)

Here, \( a_1, a_2, a_3, c_1, c_2 \) and \( c_3 \) are constants of integration. The last two of these solutions are for constant \( q \) and hence corresponds to the solutions under CDP ansatz. In this paper, we consider only the first part i.e.

\[ a = a_1 e^{\frac{2}{\sqrt{m^2-2c_1 k}}} \arctanh \left( \frac{k t - m}{\sqrt{m^2-2c_1 k}} \right). \]

(14)

For compatibility with the observed universe, we consider the solution for \( k > 0 \) and \( m > 0 \) and omit the integration constant \( c_1 \). Under the above considerations, (14) further reduces to

\[ a = a_1 e^{\frac{2}{k t} \arctanh \left( \frac{kt - m}{k} \right)} \]

(15)

In this paper, we consider a special form of the gauge function as

\[ \phi = \alpha t^\frac{1}{2} \]

(16)

where \( \alpha \) is a constant.

Substituting (11), (15), (16) in (8) and (9), we get the expressions for energy density \( \rho \), pressure \( p \) and coefficient of viscosity \( \eta \) as

\[ \rho = \frac{3}{G} \left[ \frac{4}{(kt^2 - 2mt)^2} + \frac{\kappa}{a^2} e^{-\frac{2}{k t} \arctanh \left( \frac{kt - m}{k} \right)} \right] - \frac{2}{kt^2 - 2mt} \frac{1}{4t^2} \]

(17)

\[ p = \frac{3\gamma}{G} \left[ \frac{4}{(kt^2 - 2mt)^2} + \frac{\kappa}{a^2} e^{-\frac{2}{k t} \arctanh \left( \frac{kt - m}{k} \right)} \right] - \frac{2}{kt^2 - 2mt} \frac{1}{4t^2} \]

(18)

\[ \eta = \frac{1}{3GH} \left[ \frac{8(1 + kt - m)}{(kt^2 - 2mt)^2} + \frac{(3\gamma + 1)\kappa}{a^2} e^{-\frac{2}{k t} \arctanh \left( \frac{kt - m}{k} \right)} \right] - \frac{(\gamma + 4)}{l(kt^2 - 2mt)} \frac{1}{4t^2} \frac{1}{2} \]

(19)
Fig. 1 shows variation of \( \rho \) against \( t \) for \( m = 1, G = 1, a_1 = 1/100, k = 1, t_{\text{end}} = 2m/k \approx 2 \) and various values of \( \kappa \) (=-1, 0, 1) viscous fluid. Fig. 2 shows variation of \( \eta \) against \( t \) for \( m = 1, G = 1, k = 1, a_1 = 1/100, \kappa = 1, \)
\[ t_{\text{end}} = 2m/k \approx 2 \] and various values of \( \gamma(= 1, -1, 1/3) \) in viscous fluid model.

For perfect fluid, one has \( \eta = 0 \). Then one obtains the same expressions for pressure \( p \) and energy density \( \rho \) as in viscous model.

4. Physical behaviors of the model and discussion

From eq. (17)-(19), it is observed that coefficient of bulk viscosity \( \eta \) and energy density are decreasing function of time. We have the Hubble parameter of the universe as
\[ H = \frac{\dot{a}}{a} = -\frac{2}{kt^2}. \]
In our model, the universe has finite lifetime. It starts with a big bang at \( t_0 = 0 \) and ends at \( t_{\text{end}} = \frac{2m}{k} \). Both of the energy density of the fluid and the scale factor diverge in finite time as \( t \to t_{\text{end}} \). This is the big rip behavior first suggested by Caldwell et al. (2003). We also observed that the universe begins with \( q_0 = m - 1 \), enters into the accelerating phase \( (q < 0) \) at \( t_a = \frac{m-1}{k} \), enters into super-exponential phase \( (q < -1) \) at \( t_{\text{sc}} = \frac{m}{k} \) and ends with \( q_{\text{end}} = -m - 1 \).

Now, to demonstrate how LVDP supports the observed kinematics of the universe and makes additional predictions, we plot the energy density with time. We also plot the coefficient of bulk viscosity versus time for spatially closed models of \( (\gamma = 1, -1, 1/3) \) as in Akarsu and Deren (2012).

In Fig. 1, we plot the energy density of the fluid \( \rho \) versus time \( t \) for viscous model. One may observed that the spatially open model is not possible since the positivity condition of the energy density is violated but the spatially closed and flat models are possible since the positivity condition of the energy density is satisfied. For the spatially closed and flat models, the energy density of the fluid diverges at the beginning and end of the universe. Both the scale factor and the energy density of the fluid diverges at \( t_{\text{end}} \); then we say that the universe ends with a big rip. The open models are unphysical. In Fig. 2, we plot the coefficient of bulk viscosity \( \eta \) against the time \( t \) in viscous fluid model for various values of \( \gamma(= 1, -1, 1/3) \). It is observed that \( \eta \) is a decreasing function of time for radiation dominated model \( (1/3) \) and stiff fluid matter \( (\gamma = 1) \) which is in accordance with the present observation. It is observed that false vacuum model \( \gamma = -1 \) doesn’t satisfy the present observation as \( \eta \) is a increasing function of time.

5. Conclusion:

Many authors have discussed cosmological models with a bulk viscosity. The effect of bulk viscosity is to produce a change in the perfect fluid. In this paper, we have obtained cosmological models with bulk viscosity by considering a linearly variable deceleration parameter in a scale covariant theory of gravitation proposed by Canuto et al. (1977). The nature of the energy density \( \rho \), pressure and coefficient of bulk viscosity \( \eta \) has been investigated for radiation dominated, stiff fluid model and false vacuum model has been investigated. Perfect fluid model is also investigated by considering \( \eta = 0 \) which gives the same values of pressure \( p \) and energy density \( \rho \) as in viscous fluid. From the Fig. 1, we observed that spatially open model is not possible. we observed that the radiation dominated \( (\gamma = 1/3) \) and stiff fluid models \( (\gamma = 1) \) are possible only for spatially closed universe. The big rip predicted in our models is not rule out by observations. The model also shows initial singularity and finite lifetime with \( t_{\text{end}} = \frac{2m}{k} \). The coefficient of bulk viscosity \( \eta \) decreases with increase in the age of universe for the radiation dominated \( (\gamma = 1/3) \) and stiff fluid \( (\gamma = 1) \) models. Hence the present model, despite its simplicity, might lead to a better understanding of the universe.

Acknowledgement: The Author would like to express his gratitude to Prof. N. Ibotombi Singh, Manipur University for his valuable discussion and constructive comments.
References

Beesham, A.: Class. Quantum Grav.:3,481(1986)
Dirac, P.A.M Proc.R. Soc. A333, 403 (1973)