SOME STRONGER FORMS OF $rg^{**}b^μ$ –CONTINUOUS FUNCTIONS

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Abstract: The aim of this paper is to introduce new class of functions called strongly $rg^{**}b^μ$-closed map, strongly $rg^{**}b^μ$-continuous, perfectly $rg^{**}b^μ$-continuous and strongly $rg^{**}b^μ$-irresolute functions in supra topological spaces. Some properties and characterizations of these types of functions are discussed. Also we investigate the relationship between these classes of functions.

Keywords: strongly $rg^{**}b^μ$-closed map, strongly $rg^{**}b^μ$-continuous, perfectly $rg^{**}b^μ$-continuous and strongly $rg^{**}b^μ$-irresolute functions.

1.INTRODUCTION


In this paper we introduce and investigate notions of new classes of functions namely strongly $rg^*b^μ$-closed, strongly $rg^{**}b^μ$-closed, strongly $rg^{**}b^μ$-continuous, strongly $rg^{**}b^μ$-irresolute, and almost $rg^*b^μ$-irresolute and almost $rg^{**}b^μ$-irresolute functions in supra topological spaces. Relationships between these types of functions and other classes of functions are obtained. We also note that the class of $rg^*b^μ$-closed map is properly placed between strongly $rg^{**}b^μ$-closed map and almost $rg^*b^μ$-closed map.

2. PRELIMINARIES

Definition:2.1[7]
A subclass $τ^∗ \subseteq P(X)$ is called a supra topology on X if $X \in τ^∗$ and $τ^∗$ is closed under arbitrary union. ($X,τ^∗$) is called a supra topological space (or supra space). The members of $τ^∗$ are called supra open sets.

Definition:2.2[7]
The supra closure of a set A is defined as $cls^μ (A)= \cap \{B: B \text{ is supra closed and } A \subseteq B\}$
The supra interior of a set A is defined as $Ints^μ (A)= \cup \{B: B \text{ is supra open and } A \supseteq B\}$

Definition 2.3:
Let $(X,μ)$ be a supra topological space. A subset A of X is called

(i) supra regular generalized star star b-closed set[6] (briefly $rg^{**}b^μ$-closed set) if $rg^*bcls^μ (A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in X.
Theorem: 3.2

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then $f$ is strongly $b^\ast$-closed if for every supra closed set $F$ in $X$, $f(F)$ is supra closed in $Y$.

Definition 3.3: A map $f : (X, \mu) \rightarrow (Y, \lambda)$ is said to be

(i) **strongly $b^\ast$-closed** if for every $b^\ast$-closed set $F$ in $X$, $f(F)$ is $b^\ast$-closed in $Y$.

(ii) **strongly $g^\ast b^\ast$-closed** if for every $g^\ast b^\ast$-closed set $F$ in $X$, $f(F)$ is $g^\ast b^\ast$-closed in $Y$.

(iii) **almost $g^\ast b^\ast$-closed** if for every supra regular closed set $F$ in $X$, $f(F)$ is $g^\ast b^\ast$-closed in $Y$.

Example 3.4:

Let $X = \{a, b, c, d\} = Y = Z, \mu_1 = \{\emptyset, \{a, d\}, \{a, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{a, d\}, \{a, c\}\}$. Let $f : (X, \mu_1) \rightarrow (Y, \mu_2)$ be defined by $f(a) = a, f(b) = b, f(c) = c, f(d) = a$ and $g : (Y, \mu_2) \rightarrow (Z, \mu_3)$ be an identity map. Then $f$ is strongly $g^\ast b^\ast$-closed but $g$ is supra closed map.

Theorem: 3.5

(i) Every strongly $g^\ast b^\ast$-closed map is almost $g^\ast b^\ast$-closed map.

(ii) Every strongly $g^\ast b^\ast$-closed map is $g^\ast b^\ast$-closed map.

(iii) Every $g^\ast b^\ast$-closed map is $b^\ast$-closed map.

Proof: It is obvious.

Remark: 3.6

The converse of the above theorem is not true and it is shown by the following example.

Example 3.7:

Let $X = \{a, b, c, d\} = Y, \mu_1 = \{\emptyset, \{a, d\}, \{a, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{a, d\}, \{a, c\}\}$. Let $f : (X, \mu_1) \rightarrow (Y, \mu_2)$ be the identity map. Here $f$ is almost $g^\ast b^\ast$-closed but not strongly $g^\ast b^\ast$-closed, since $f(\{b, d\}) = \{b, d\}$ is not $g^\ast b^\ast$-closed in $(Y, \mu_2)$.

Example 3.8:

Let $X = \{a, b, c, d\} = Y, \mu_1 = \{\emptyset, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{a, d\}, \{a, c\}\}$. Let $f : (X, \mu_1) \rightarrow (Y, \mu_2)$ be the identity map. Here $f$ is $g^\ast b^\ast$-closed but not strongly $g^\ast b^\ast$-closed, since $f(\{a, b, c\}) = \{a, b, c\}$ is not $g^\ast b^\ast$-closed in $(Y, \mu_2)$.

Example 3.9:

Let $X = \{a, b, c, d\} = Y, \mu_1 = \{\emptyset, \{a, b\}, \{a, d\}, \{a, c\}, \{a, b, c\}\}$ and $\mu_2 = \{\emptyset, \{a, b\}, \{a, d\}, \{a, c\}\}$. Let $f : (X, \mu_1) \rightarrow (Y, \mu_2)$ be the identity map. Here $f$ is almost $g^\ast b^\ast$-closed but not $g^\ast b^\ast$-closed, since $f(\{b, c, d\}) = \{b, c, d\}$ is not $g^\ast b^\ast$-closed in $(Y, \mu_2)$.
If \( f : (X, \tau) \to (Y, \sigma) \) is almost \( g^**b^\mu \)-closed map and \( g : (Y, \sigma) \to (Z, \gamma) \) is strongly \( r^g**b^\mu \)-closed map then \( g \circ f : X, \tau \to (Z, \gamma) \) is almost \( r^g**b^\mu \)-closed map.

**Proof:** It is obvious.

**Theorem 3.11**
The composite mapping of two strongly \( r^g**b^\mu \)-closed map is strongly \( r^g**b^\mu \)-closed map.

From the above theorem and example we have the following diagram

![Diagram](image)

**4. STRONGLY \( r^g**b^\mu \)-CONTINUOUS AND PERFECTLY \( r^g**b^\mu \)-CONTINUOUS MAPS**

**Definition 4.1**
A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be

- (i) **strongly \( r^g**b^\mu \)-continuous** if the inverse image of every \( r^g**b^\mu \)-open set of \( Y \) is supra open in \( (X, \tau) \).
- (ii) **perfectly \( b^\mu \)-continuous** if the inverse image of every supra-open set of \( Y \) is \( cl^\mu(open^\mu) \) in \( (X, \tau) \).
- (iii) **perfectly \( r^g**b^\mu \)-continuous** if the inverse image of every \( r^g**b^\mu \)-open set of \( Y \) is \( cl^\mu(open^\mu) \) in \( (X, \tau) \).

**Theorem 4.2:**
(i) Every strongly \( r^g**b^\mu \)-continuous function is supra-continuous.
(ii) If a function \( f : (X, \tau) \to (Y, \sigma) \) is said to be perfectly \( r^g**b^\mu \)-continuous function then \( f \) is perfectly \( b^\mu \)-continuous.
(iii) If a function \( f : (X, \tau) \to (Y, \sigma) \) is said to be perfectly \( r^g**b^\mu \)-continuous function then \( f \) is strongly \( r^g**b^\mu \)-continuous.

**Remark:**
The converse of the above theorem need not be true as shown in the example below.

**Example 4.3:**
(i) \( X=\{a,b,c,d\} \subset Y, \mu_1 =X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\} \). Let \( f : (X, \mu_1) \to (Y, \mu_2) \) be defined by \( f(a)=a, f(b)=b, f(c)=d, f(d)=c \). Here \( f \) is supra continuous but not strongly \( r^g**b^\mu \)-continuous since \( f^{-1}([b,c,d])=[b,c,d] \) is not supra open in \( (X, \mu_1) \).

(ii) \( X=\{a,b,c,d\} \subset Y, \mu_1 =X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\} \). Let \( f : (X, \mu_1) \to (Y, \mu_2) \) be the identity map. Here \( f \) is perfectly supra continuous but not perfectly \( r^g**b^\mu \)-continuous since \( f^{-1}([b,c,d])=[b,c,d] \) is not supra open in \( (X, \mu_1) \).

(iii) \( X=\{a,b,c,d\} \subset Y, \mu_1 =X, \emptyset, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\} \). Let \( f : (X, \mu_1) \to (Y, \mu_2) \) be the identity map. Here \( f \) is strongly \( r^g**b^\mu \)-continuous but not perfectly \( r^g**b^\mu \)-continuous since \( f^{-1}([c,a,d])=[c,a,d] \) is not supra closed in \( (X, \mu_1) \).

**Theorem 4.4**
If \( f : (X, \tau) \to (Y, \sigma) \) is strongly \( r^g**b^\mu \)-continuous and \( g : (Y, \sigma) \to (Z, \gamma) \) is \( r^g**b^\mu \)-continuous then \( g \circ f : (X, \tau) \to (Z, \gamma) \) is supra continuous.

**Proof:** Obvious.

**5. STRONGLY \( r^g**b^\mu \)-IRRESOLUTE AND ALMOST \( r^g**b^\mu \)-IRRESOLUTE FUNCTIONS**

**Definition 5.1**
A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be

- (i) **strongly \( r^g**b^\mu \)-irresolute** if \( f^{-1}(V) \) is supra open in \( (X, \tau) \) for every \( r^g**b^\mu \)-open set \( V \) of \( (Y, \sigma) \).
- (ii) **strongly \( r^g**b^\mu \)-irresolute** if \( f^{-1}(V) \) is supra open in \( (X, \tau) \) for every \( r^g**b^\mu \)-open set \( V \) of \( (Y, \sigma) \).
- (iii) **strongly \( b^\mu \)-irresolute** if \( f^{-1}(V) \) is supra open in \( (X, \tau) \) for every \( b^\mu \)-open set \( V \) of \( (Y, \sigma) \).
- (iv) **almost \( r^g**b^\mu \)-irresolute** if \( f^{-1}(V) \) is \( b^\mu \)-open in \( (X, \tau) \) for every \( r^g**b^\mu \)-open set \( V \) of \( (Y, \sigma) \).
- (v) **almost \( r^g**b^\mu \)-irresolute** if \( f^{-1}(V) \) is \( b^\mu \)-open in \( (X, \tau) \) for every \( r^g**b^\mu \)-open set \( V \) of \( (Y, \sigma) \).
Theorem: 5.2
(i) Every strongly $rg^{**}b^{u}$-irresolute function is $rg^{**}b^{u}$-irresolute.
(ii) Every strongly $rg^{**}b^{u}$-irresolute function is $rg^{**}b^{u}$-continuous.
(iii) Every strongly $rg^{**}b^{u}$-irresolute function is $rg^{**}b^{u}$-irresolute.
(iv) Every $rg^{**}b^{u}$-irresolute function is $rg^{**}b^{u}$-continuous.
(v) $f:(X,\tau)ightarrow(Y,\sigma)$ is strongly $rg^{**}b^{u}$-irresolute function then $f$ is strongly $b^{u}$-irresolute.
(vi) $f:(X,\tau)ightarrow(Y,\sigma)$ is strongly $rg^{**}b^{u}$-irresolute function then it is strongly $b^{u}$-irresolute.
(vii) $f:(X,\tau)ightarrow(Y,\sigma)$ is almost $rg^{**}b^{u}$-irresolute then it is $b^{u}$-continuous map.
(viii) If $f:(X,\tau)ightarrow(Y,\sigma)$ is almost $rg^{**}b^{u}$-irresolute then it is $rg^{**}b^{u}$-irresolute map.
(ix) If $f:(X,\tau)ightarrow(Y,\sigma)$ is almost $rg^{**}b^{u}$-irresolute then it is $b^{u}$-continuous map.
(x) If $f:(X,\tau)ightarrow(Y,\sigma)$ is almost $rg^{**}b^{u}$-irresolute then it is almost $rg^{**}b^{u}$-irresolute map.
(xi) If $f:(X,\tau)ightarrow(Y,\sigma)$ is $b^{u}$-continuous then it is $rg^{**}b^{u}$-continuous map.

Proof: Obvious.

Remark 5.3:
The converse of the above theorem need not be true as shown in the examples below.

Example 5.4:
(i) $X=\{a,b,c,d\}=\{a\}$, $\mu_{1}=[X,\emptyset,\{a\},\{\emptyset\},\{a\},\emptyset,\{a\},\emptyset,\{a\},\emptyset]$, $\mu_{2}=[Y,\emptyset,\{a\},\{a\},\emptyset,\{a\},\emptyset,\{a\},\emptyset]$, $\mu_{1}$ is strongly $b^{u}$-irresolute but not strongly $rg^{**}b^{u}$-irresolute, since $f^{-1}\subseteq(\{a\})=[a,c,d]$ is not supra open in $(X,\mu_{1})$.

(ii) $X=\{a,b,c,d\}=\{a\}$, $\mu_{1}=[X,\emptyset,\{a\},\{\emptyset\},\{a\},\emptyset,\{a\},\emptyset,\{a\},\emptyset]$, $\mu_{2}=[Y,\emptyset,\{a\},\{a\},\emptyset,\{a\},\emptyset,\{a\},\emptyset]$, $\mu_{1}$ is strongly $b^{u}$-irresolute but not strongly $rg^{**}b^{u}$-irresolute, since $f^{-1}\subseteq(\{a\})=[a,c,d]$ is not supra open in $(X,\mu_{1})$.

(iii) $X=\{a,b,c,d\}=\{a\}$, $\mu_{1}=[X,\emptyset,\{a\},\{\emptyset\},\{a\},\emptyset,\{a\},\emptyset,\{a\},\emptyset]$, $\mu_{2}=[Y,\emptyset,\{a\},\{a\},\emptyset,\{a\},\emptyset,\{a\},\emptyset]$, $\mu_{1}$ is strongly $b^{u}$-irresolute but not strongly $rg^{**}b^{u}$-irresolute, since $f^{-1}\subseteq(\{a\})=[a,c,d]$ is not supra open in $(X,\mu_{1})$.

From the above theorem and above examples we have the following implications:
Theorem: 5.5
Let \((X, \tau) \rightarrow (Y, \sigma) \rightarrow (Z, \gamma)\) be any two functions then the composition \(g \circ f : X, \tau \rightarrow (Z, \gamma)\) is
i) almost \(rg^*b^\mu\)-irresolute if \(f\) is almost \(rg^*b^\mu\)-irresolute and \(g\) is \(rg^*b^\mu\)-irresolute
ii) almost \(rg^*b^\mu\)-irresolute if \(f\) is \(b^\mu\)-irresolute and \(g\) is almost \(rg^*b^\mu\)-irresolute.

Proof: It is obvious.

REFERENCES:


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