DOMINATION IN PRODUCT OF CUBIC FUZZY GRAPHS

M. Vijaya and a. Kannan

1. Head P.G. and Research Department of Mathematics, Marudupandiyar College, Vallam, Thanjavur, Tamil Nadu, India.
E-Mail: mathvijaya79@yahoo.com
2. Research Scholar, P.G. and Research Department of Mathematics, Marudupandiyar College, Vallam, Thanjavur, Tamil Nadu, India.
E-mail: kannanarumugam1980@gmail.com

Abstract: In this paper to generalize the domination in product of cubic fuzzy graphs. We discuss the concept of domination, total domination and product of fuzzy graphs. We determine some result on domination in product of cubic fuzzy graphs.

Keywords: Fuzzy graph, Domination, Total domination, product of fuzzy graph, cubic fuzzy graph.

1. INTRODUCTION

The theory of domination is formalized by Clauge Berge in his book “Theory of graphs and its applications.” Berge mentions the strategies of keeping a number of locations under surveillance, by a set of radar station. Oystein ore was a first person to use the term domination number in his book on graph theory. The domination number is introduced by E.J.Cockayane and S.T.Hedetniemi Rosenfeld introduced the notation of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A.Somasundaram and S.Somasundaram discussed domination in fuzzy graphs they defined dominating using effective edges in fuzzy graph. A.Nagoorgani and V.T.Chandrasekaran discussed domination in fuzzy graphs using strong arc. We also discuss the domination, total domination in product of cubic fuzzy graphs.

2. PRELIMINARIES

Definition 2.1 (Fuzzy graph)

A fuzzy graph \( G(\sigma, \mu) \) is a pair of function \( \sigma : V \rightarrow [0,1] \) and \( \mu : V \times V \rightarrow [0,1] \) such that \( \sigma(u, v) \leq \sigma(u) \land \sigma(v) \) for all \( u, v \) in \( V \).

Example 2.1

A fuzzy graph \( G = (\sigma, \mu) \) where \( \sigma = \left\{ \begin{array}{l} 0.5, 0.4, 0.6 \\ 0.3, 0.4, 0.2 \end{array} \right\} \) and \( \mu = \left\{ \begin{array}{l} u, v \\ u, w \\ v, w, u \end{array} \right\} \).

Figure 1

Definition 2.2 (Fuzzy Domination Number)

Let \( G = (V, \sigma, \mu) \) be a fuzzy graph. Then \( D \subseteq V \) is said to be a fuzzy dominating set of \( G \) if for every \( V \in V - D \) there exists \( u \) in \( D \) such that \( \mu(u, v) = \sigma(u) \land \sigma(v) \). The minimum scalar cardinality of \( D \) is called the fuzzy domination number and is denoted by \( \gamma(G) \).
Example 2.2

\[ D = \{v_1\} \]
\[ \gamma(G) = 0.4 \]

Figure 2

Definition 2.3 (Fuzzy Total Domination Number)

A dominating set is said to be total dominating set if every vertex in \( V \) is effectively adjacent to at least one vertex in \( S \). The minimum cardinality taken over all minimal total dominating set is called the total domination number and is denoted by \( \gamma_{ft}(G) \).

Example 2.3

\[ \gamma_{ft}(G) = \{S, t\} = 2. \]

Definition 2.4 (Product of fuzzy graph)

Let \( G \) be a graph whose vertex set is \( V \), \( \mu \) be a fuzzy subset of \( V \) and \( \rho \) be a fuzzy subset of \( V \times V \) we call \((\mu, \rho)\) a product partial fuzzy subgraph of \( G \) (in short a product fuzzy graph) if \( \rho(x, y) \leq \mu(x) \times \mu(y) \) for all \( x, y \in V \).

Example 2.4

Let \( V = \{a, b, c\} \), \( \mu \) be the fuzzy subset of \( V \) defined as \( \mu(a) = 1/4, \mu(b) = 1/2 \) and \( \mu(c) = 3/4 \). Let \( \rho \) be the fuzzy subset of \( V \times V \) defined as \( \rho(a, b) = 1/10, \rho(b, c) = 2/8 \) and \( \rho(a, c) = 2/16 \) \((\mu, \rho)\) is a product fuzzy graph.

Definition 2.5 (Domination in product of fuzzy graph)

Let \( G = (v, \mu, \rho) \) be a product fuzzy graph and \( u, v \in V \), then we say \( u \) dominates \( v \) if and only if \( \rho(u, v) = \mu(u) \times \mu(v) \)

Example 2.5

Consider a fuzzy graph \( G = (v, \mu, \rho) \), where \( V = \{v_1, v_2, v_3, v_4, v_5\} \), \( \mu(v_1) = 0.3, \mu(v_2) = 0.6, \mu(v_3) = 0.8, \mu(v_4) = 0.9 \), and \( \mu(v_5) = 0.8 \), and \( \rho(v_1, v_2) = 0.3, \rho(v_2, v_3) = 0.5, \rho(v_2, v_4) = 0.4, \rho(v_3, v_4) = 0.8, \rho(v_3, v_5) = 0.8, \rho(v_4, v_3) = 0.6, \rho(v_5, v_1) = 0.2 \). Here \( v_3 \) dominates \( v_4 \) and \( v_5 \), and \( v_2 \) dominates \( v_1 \). Clearly, \( S = \{v_1, v_3\} \subset V \), is the minimum dominating set of \( G \), and therefore \( \gamma(G) = 1.1 \).

Definition 2.6 (Cubic Fuzzy Graph)

A three regular fuzzy graph is called cubic fuzzy graph (i.e.) fuzzy graph whose vertices all have degree 3.

Example 2.6
Definition 2.7 (Domination in cubic fuzzy graph)

Let $G$ be a fuzzy graph; and the domination number $\gamma(G)$ of every cubic $n$–vertex graph $G$ with minimum degree atleast 3. A cubic graph $G$ has $\text{mr}(G) = 3$. A three regular fuzzy graph whose vertices degree 3.

3. MAIN RESULTS

Theorem 3.1

For any product of cubic fuzzy graph $G = (v, \mu, \rho)$ we have $\gamma(G) \leq \rho$. Further the equality holds if $\rho(uv) < \mu(u) \times \mu(v) \forall \ u, v \in V$.

Proof

Let $D$ be a dominating set of $G$, then $\forall \ v \in V - D, \exists u \in D$ such that $\rho(uv) = \mu(u) \times \mu(v)$ therefore $|D| \leq |V|$ and hence $\gamma(G) \leq \rho$. Now if $\rho(uv) < \mu(u) \times \mu(v) < \mu(u) \times \mu(v) \forall \ u, v \in V$. Then $V$ is independent vertex set and every vertex $v \in V$ is an isolated vertex. Hence the dominating set $D$ must contain all isolated vertex in $G$ then $D = V$ and hence $\gamma(G) = \rho$.

Theorem 3.2

For any connected cubic fuzzy graph $G$ with $P \geq 3$ vertices and exactly one vertex has $\Delta_N(G) \leq P - 2$ the triple connected dominating vertices is 3.

Proof

Let $G$ be a connected cubic fuzzy graph with vertices $P \geq 3$ exactly one vertex has maximum neighbourhood degree $\Delta_N(G) \leq P - 2$. Let $V$ be the vertex of maximum neighbourhood degree $\Delta_N(G) \leq P - 2$. Let $v_1, v_2, \ldots, v_{P-2}$ be the vertices which are adjacent to $V$. Let $V_p - 1$ be the vertex which is not adjacent to $V$. Since $G$ is connected $V_{p-1}$ is adjacent to a vertex $V_i$ for some $i$. Then $S = \{V_j, V_k, V_{p-1}\}$ from a minimum triple connected domination set of $G$.

Theorem 3.3

For any product of cubic fuzzy graph $G = (V, \mu, \rho)$, $\gamma_t(G) = \rho$ if and only if every vertex of $G$ has unique neighbour.

Proof

If every vertex of a product of cubic fuzzy graph $G$ has unique neighbour then $V$ is the only total dominating set of $G$ so that $\gamma_t(G) = \rho$ conversely suppose $\gamma_t(G) = \rho$. If there exists a vertex $V$ with two neighbours $x$ and $y$ then $V - \{x\}$ is a total dominating set of $G$ so that $\gamma_t(t) < \rho$ which is a contradiction.

Example 2.6

Consider a cubic fuzzy graph shown in the figure.

$$V = \{x, y, z, w\} \ \mu(x) = 0.2 \ \mu(y) = 0.3 \ \mu(2) = 0.4 \ \mu(w) = 0.5$$
$D_1 = \{z\}$ with fuzzy cardinality.

$$|D_1| = \sum \mu(V) \quad V \in D_1 = \mu(z) = 0.4$$

$D_2 = \{y, z\}$ with fuzzy cardinality.

$$|D_2| = \sum \mu(V) \quad V \in D_2 = \mu(y) + \mu(z) = 0.3 + 0.4 = 0.7$$

$D_3 = \{y, w\}$ with fuzzy cardinality.

$$|D_3| = \sum \mu(V) \quad V \in D_3 = \mu(y) + \mu(w) = 0.3 + 0.5 = 0.8$$

$D_4 = \{x, z, w\}$ with fuzzy cardinality.

$$|D_4| = \sum \mu(V) \quad V \in D_4 = \mu(x) + \mu(y) + \mu(z) = 0.2 + 0.3 + 0.4 = 0.9$$

$D_5 = \{x, z, w\}$ with fuzzy cardinality.

$$|D_5| = \sum \mu(V) \quad V \in D_5 = \mu(x) + \mu(z) + \mu(w) = 0.2 + 0.4 + 0.5 = 1.1$$

$D_6 = \{x, y, z, w\}$ with fuzzy cardinality.

$$|D_6| = \sum \mu(V) \quad V \in D_6 = \mu(x) + \mu(y) + \mu(z) + \mu(w) = 0.2 + 0.3 + 0.4 + 0.5 = 1.4$$

The dominate number of $G$ is

$$\gamma(G) = \min\{0.4, 0.7, 0.8, 0.9, 1.1, 1.4\} = 0.4$$

4. CONCLUSION

In this paper we define the concept of domination, total domination fuzzy graphs. Next we discuss product of fuzzy graphs and further we proved theorem based on domination in product of cubic fuzzy graphs.

5. REFERENCES


