VISCO-ELASTIC MHD CONVECTIVE PERIODIC FLOW THROUGH POROUS MEDIUM IN A ROTATING VERTICAL CHANNEL WITH THERMAL RADIATION

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Abstract: An analytical study of magnetohydrodynamic (MHD) mixed convection flow of a viscoelastic, incompressible and electrically conducting fluid through a porous medium filled in a vertical channel is carried out. A magnetic field of uniform strength is applied perpendicular to the planes of the channel walls. The fluid is acted upon by periodic time variation of the pressure gradient in the vertically upward direction. The temperature of one of the plates is non-uniform and the temperature difference of the walls of the channel is high enough to induce heat transfer due to radiation. The fluid and the channel rotate in unison with an angular velocity about the axis normal to the plates of the channel. An exact analytical solution of the problem is obtained. Two cases of small and large rotations have been considered to assess the effects of different parameters involved in the flow problem. The velocity field, the amplitude and the phase angle of the skin friction are shown graphically and discussed in detail.

Keywords: Visco-elastic, periodic, convection, magnetohydrodynamic (MHD), rotating, heat radiation.

INTRODUCTION

The hydrodynamic rotating flow of electrically conducting viscoelastic incompressible fluids has gained considerable attention because of its numerous applications in cosmical and geophysical fluid dynamics. In geophysics it is applied to measure and study the positions and velocities with respect to a fixed frame of reference on the surface of earth which rotate with respect to an inertial frame in the presence of its magnetic field. The subject of geophysical dynamics nowadays has become an important branch of fluid dynamics due to the increasing interest to study environment. In astrophysics it is applied to study the stellar and solar structure, inter planetary and inter stellar matter, solar storms and flares etc. During the last few decades it also finds its application in engineering. Among the applications of rotating flow in porous media to engineering disciplines, one can find the food processing industry, chemical process industry, centrifugation and filtration processes and rotating machinery. In recent years a number of studies have also appeared in the literature on the fluid phenomena on earth involving rotation to a greater or lesser extent viz. Vidyanidhu and Nigam [1] Gupta [2] Jana and Datta [3], Mazumder [4] obtained an exact solution of an oscillatory Couette flow in a rotating system. Thereafter Ganapathy [5] presented the solution for rotating Couette flow. Singh [6] analyzed the oscillatory magnetohydrodynamic (MHD) Couette flow in a rotating system in the presence of transverse magnetic field. Singh [7] also obtained an exact solution of magnetohydrodynamic (MHD) mixed convection flow in a rotating vertical channel with heat radiation. The study of the flows of visco-elastic fluids is important in the fields of petroleum technology and in the purification of crude oils. In recent years, flows of visco-elastic fluids attracted the attention of several scholars in view of their practical and fundamental importance associated with many industrial applications. Literature is replete with the various flow problems considering variety of geometries such as Rajgopal [8-9], Rargopal and Gupta [10-11], Ariel [12], Pop and Gorla [13]. Hayat et al [14] discussed periodic unsteady flows of a non-Newtonian fluid. Choudhury and Das [15] studied the oscillatory viscoelastic flow in a channel filled with porous medium in the presence of radiative heat transfer. Singh [16] analyzed viscoelastic mixed convection MHD oscillatory flow through a porous medium filled in a vertical channel. Taking the rotating frame of reference into account Puri [17] investigated rotating flow of an elastic-viscous fluid on an oscillating plate. Puri and Kulshrestha [18] analyzed rotating flow of non-Newtonian fluids. Rajgopal [19] investigated flow of viscoelastic fluids between rotating disks. Applying quasi-linearization to the problem Verma et al [20] analyzed steady laminar flow of a second grade fluid between two rotating porous disks. Hayat et al [21] studied fluctuating flow of a third order fluid on a porous plate in a rotating medium. Hayat et al [22] investigated the unsteady hydromagnetic rotating flow of a conducting second grade fluid.

The purpose of the present analysis is to study a periodic mixed convection flow of an electrically conducting viscoelastic incompressible fluid flow through a porous medium in a vertical channel. The entire system rotates about an axis perpendicular to the planes of the plates of the channel and a uniform magnetic field is also applied along this axis of rotation. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field is neglected. The non-uniform temperature difference of the channel walls is high enough to induce the thermal radiations.
FORMULATION OF THE PROBLEM

Consider the flow of a viscoelastic, incompressible and electrically conducting fluid in a rotating vertical channel as shown in Fig.1.

\[ z^* = -\frac{d}{z}, u^* = v^* = 0, T^* = 0. \]
\[ z^* = \frac{d}{z}, u^* = v^* = 0, T^* = T_0 \cos \omega^* t^*. \]

In order to derive basic equations for the problem under consideration following assumptions are made:

(i) The flow considered is unsteady and laminar.
(ii) The fluid is finitely conducting and with constant physical properties.
(iii) A magnetic field of uniform strength is applied normal to the flow.
(iv) The magnetic Reynolds number is taken to be small enough so that the induced magnetic field is neglected.
(v) Hall effect, electrical and polarization effects are neglected.
(vi) It is assumed that the fluid is optically thin with relatively low density.
(vii) The entire system (consisting of channel plates and the fluid) rotates about an axis perpendicular to the plates.
(viii) Since plates are infinite so all physical quantities except pressure depend only on \( z^* \) and \( t^* \).

Under these assumptions, we write hydromagnetic equations of continuity, motion and energy in a rotating frame of reference as:

\[ \nabla \cdot V = 0. \]  
\[ \frac{\partial V}{\partial t} + (V \cdot \nabla) V + 2\Omega \times V = \frac{-1}{\rho} \nabla P + \frac{\partial_t}{\partial t} V + \frac{\partial_t}{\partial z} V + \frac{1}{\rho} (J \times B) + F, \]  
\[ \rho C_p \left[ \frac{\partial T}{\partial z} + (\nabla T) V \right] = k \nabla^2 T^* - \nabla q. \]

In equation (2) the last term on the left hand side is the Coriolis force. On the right hand side of (2) the last term \( F(=g\beta T^*) \) accounts for the force due to buoyancy and the second last term is the Lorentz force due to magnetic field \( B \) and is given by
\[ J \times B = \sigma (V \times B) \times B. \] (4)

In the term third from last of equation (2), \( \mathcal{E} \) is the Cauchy stress tensor and the constitutive equation derived by Coleman and Noll [23] for an incompressible homogeneous fluid of second order is

\[ \mathcal{E} = -p I + \mu_1 A_1 + \mu_2 A_2 + \mu_3 A_3^2. \] (5)

Here \(-p I\) is the interdeterminate part of the stress due to constraint of incompressibility, \(\mu_1, \mu_2\) and \(\mu_3\) are the material constants describing viscosity, elasticity and cross-viscosity respectively. The kinematic \(A_1\) and \(A_2\) are the Rivelen Ericson constants defined as

\[ A_1 = (\nabla \mathcal{E}) + (\nabla \mathcal{E})^T, \quad A_2 = \frac{dA_1}{dt} + (\nabla \mathcal{E})^T A_1 + A_1 (\nabla \mathcal{E}). \] where \(\nabla\) denotes the gradient operator and \(d/dt\) the material time derivative. According to Markovitz and Coleman [24] the material constants \(\mu_1, \mu_3\) are taken as positive and \(\mu_2\) as negative. The modified pressure \(p^* = p' - \frac{\Omega^2}{2} |\mathbf{R}|^2\), where \(\mathbf{R}\) denotes the position vector from the axis of rotation, \(p'\) denotes the fluid pressure, \(J\) is the current density and all other quantities have their usual meanings and have been defined in the text to time.

Following Cogley et al [25] the last term in the energy equation (3) stands for the radiative heat flux, which is given by

\[ \nabla q = \frac{\partial q^*}{\partial x^*} = 4\alpha^2 T^*. \] (6)

In the present analysis we consider an unsteady flow of a viscoelastic incompressible and electrically conducting fluid bounded by two infinite insulated vertical plates distance ‘\(d\)’ apart. A coordinate system is chosen such that the \(X^*\)-axis is oriented upward along the centerline of the channel and \(Z^*\)-axis taken perpendicular to the planes of the plates lying in \(x^* = \pm \frac{d}{2}\) planes. The non-uniform temperature of the plate at \(z^* = \pm \frac{d}{2}\) is assumed to be varying periodically with time. The \(Z^*\)-axis is considered to be the axis of rotation about which the fluid and the plates are assumed to be rotating as a solid body with a constant angular velocity \(\Omega\) (0, 0, \(\Omega^*\)). A transverse magnetic field of uniform strength \(B (0, 0, B_0)\) is also applied along the axis of rotation. The velocity may reasonably be assumed with its components along \(x^*, y^*, z^*\) directions as \(V (u^*, v^*, 0)\). The equation of continuity is then satisfied identically. Using the velocity and the magnetic field distribution as stated above the magnetohydrodynamic (MHD) flow in the rotating channel is governed by the following Cartesian equations:

\[ \frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + \frac{\phi_1}{\rho} \frac{\partial^2 u^*}{\partial x^*^2} + \frac{\phi_2}{\rho} \frac{\partial^3 u^*}{\partial x^*^3} + 2\Omega^* v^* - \frac{\sigma B_0^2}{\rho K^*} u^* - \frac{\sigma B_0^2}{\rho K^*} v^* + g \beta T^*, \] (7)

\[ \frac{\partial v^*}{\partial t^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial y^*} + \frac{\phi_1}{\rho} \frac{\partial^2 v^*}{\partial y^*^2} + \frac{\phi_2}{\rho} \frac{\partial^3 v^*}{\partial y^*^3} - 2\Omega^* u^* - \frac{\sigma B_0^2}{\rho K^*} u^* - \frac{\sigma B_0^2}{\rho K^*} v^* + g \beta T^*, \] (8)

\[ 0 = -\frac{1}{\rho} \frac{\partial p^*}{\partial z^*}, \] (9)

\[ \rho c_p \frac{\partial T^*}{\partial t^*} = k \frac{\partial^2 T^*}{\partial x^*^2} - 4\alpha^2 T^*. \] (10)

where \(\rho\) is the density, \(\phi_1\) is the kinematic viscosity, \(p^*\) is the modified pressure, \(t^*\) is the time, \(\sigma\) is the electric conductivity, \(g\) is the acceleration due to gravity, \(k\) is the thermal conductivity, \(c_p\) is the specific heat at constant pressure and \(\alpha\) is the mean radiation absorption coefficient. Equation (9) shows the constancy of the hydrodynamic pressure along the axis of rotation. We shall assume now that the fluid flows under the influence of pressure gradient varying periodically with time in the \(X^*\)-axis only is of the form

\[ \frac{\partial p^*}{\rho \partial x^*} = A \cos \omega^* t^* \quad \text{and} \quad \frac{\partial p^*}{\rho \partial y^*} = 0, \] (11)

where \(A\) is a constant.

The boundary conditions for the problem are

\[ z^* = \frac{d}{2}, \quad u^* = v^* = 0, \quad T^* = T_0 \cos \omega^* t^* , \] (12)

\[ z^* = -\frac{d}{2}, \quad u^* = v^* = 0, \quad T^* = 0, \] (13)

where \(T_0\) is the mean temperature and \(\omega^*\) is the frequency of oscillations.

Introducing the following non-dimensional quantities into equations (4) and (5)

\[ \eta = \frac{x^*}{d}, \quad x = \frac{x^*}{d}, \quad y = \frac{y^*}{d}, \quad u = \frac{u^*}{U}, \quad v = \frac{v^*}{U}, \quad T = T^* / T_0, \quad t = \frac{\tau^*}{d}, \] (14)

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\[ \omega = \frac{\omega^*}{u}, \quad p = \frac{p^*}{\rho u^2}, \]  

we get
\[ Re \frac{\partial \nu}{\partial x} = -Re \frac{\partial \nu}{\partial x} + \frac{\partial^2 \nu}{\partial \eta^2} + \gamma \frac{\partial^2 \nu}{\partial \eta^2 \partial t} + 2\Omega \nu - M^2 \nu - \frac{1}{K} \nu + Gr T . \]  

\[ -Re \frac{\partial \nu}{\partial y} + \frac{\partial^2 \nu}{\partial \eta^2} + \gamma \frac{\partial^2 \nu}{\partial \eta^2 \partial t} = 2\Omega \nu - M^2 \nu - \frac{1}{K} \nu . \]  

\[ Pe \frac{\partial \Theta}{\partial t} - \frac{\partial^2 \Theta}{\partial \eta^2} - N^2 \Theta , \]  

where \( U \) is the mean axial velocity, '*' represents the dimensional physical quantities, \( Re = \frac{\nu d}{\eta} \) is the Reynolds number,
\[ \Omega = \frac{\omega^*}{\omega_1} \]  

is the rotation parameter,
\[ K = \frac{k^*}{\delta^2} \]  

is the permeability of the porous medium,

\[ M = B_0 d \frac{\delta^2}{\sqrt{\rho}} \]  

is the Hartmann number,
\[ Gr = \frac{g \delta^2 \eta_0}{\delta \eta \nu} \]  

is the Grashof number,
\[ Pe = \frac{k}{\rho \nu d \delta \nu} \]  

is the Peclet number,
\[ N = \frac{2 \eta d}{\delta \nu} \]  

is the radiation parameter.

The boundary conditions in the dimensionless form become
\[ \eta = \frac{1}{2}; \quad u = v = 0, \quad T = \cos \omega t, \]  

\[ \eta = -\frac{1}{2}; \quad u = v = 0, \quad T = 0. \]  

For the oscillatory internal flow we shall assume that the fluid flows only under the influence of a non-dimensional pressure gradient oscillating in the direction of x-axis only which is of the form
\[ -\frac{\partial \nu}{\partial x} = A \cos \omega t. \]  

**SOLUTION OF THE PROBLEM**

Now combine equations (15) and (16) into single equation by introducing a complex function of the form \( F = u + iv \), we get
\[ Re A \cos \omega t + \frac{\partial^2 F}{\partial \eta^2} + \gamma \frac{\partial^2 F}{\partial \eta^2 \partial t} - (M^2 + K^{-1} + 2i\Omega)F + Gr T, \]  

with corresponding boundary conditions as
\[ \eta = \frac{1}{2}; \quad F = 0, \quad T = \cos \omega t. \]  

\[ \eta = -\frac{1}{2}; \quad F = 0, \quad T = 0. \]  

In view of the form of the selected form of the pressure gradient in equation (20) and the linearity of the differential equations (21) and (17) to be solved under boundary conditions (22) and (23), we assume in complex notations the solution of the problem as
\[ F(\eta, t) = F_0(\eta)e^{i\omega t}, \quad T = \theta_0(\eta)e^{i\omega t} - \frac{\partial \nu}{\partial x} = A \cos \omega t = Ae^{i\omega t}. \]  

The boundary conditions (22) and (23) in complex notations can also be written as
\[ \eta = \frac{1}{2}; \quad F = 0, \quad T = e^{i\omega t}, \]  

\[ \eta = -\frac{1}{2}; \quad F = 0, \quad T = 0. \]  

Substituting these expressions (24) in equations (17) and (21), we get
\[ l^2 \frac{\partial^2 F_0}{\partial \eta^2} - m^2 F_0 = -Re A - Gr \theta_0, \]  

\[ \frac{\partial^2 \theta_0}{\partial \eta^2} - n^2 \theta_0 = 0, \]  

where \( l = \sqrt{1 + i\omega \gamma} \), \( m = \sqrt{M^2 + K^{-1} + 2i\Omega + i\omega Re} \), \( n = \sqrt{N^2 + i\omega Pe} \). The transformed boundary conditions reduce to
The ordinary differential equations (27) and (28) are solved under the boundary conditions (29) and (30) for the velocity and temperature fields. The solution of the problem is obtained as

\[
\begin{align*}
\eta = \frac{1}{2}; & \quad F_0 = 0, \quad \theta_0 = 1, \\
\eta = -\frac{1}{2}; & \quad F_0 = 0, \quad \theta_0 = 0.
\end{align*}
\]

The solution of the problem is obtained as

\[
\begin{align*}
u(\eta, t) &= \left[ \frac{ARe}{m^2} \left( 1 - \frac{\cosh \frac{\pi t}{2}}{\cosh \frac{\pi}{2t}} \right) + \frac{Gr}{(i\pi n^2 - m^2)} \left( \frac{\sinh \frac{\pi t}{2}}{\sinh \frac{\pi}{2t}} - \frac{\sinh \frac{\pi}{2}}{\sinh \frac{\pi}{t}} \right) \right] e^{i\omega t},
\end{align*}
\]

and

\[
\begin{align*}
T(\eta, t) &= \frac{\sinh n(\eta + \frac{1}{2})}{\sinh n} e^{i\omega t}.
\end{align*}
\]

The validity and correctness of the present solution is verified by taking \(\gamma = Gr = \Omega = M = 0\) i.e. for the horizontal channel filled with ordinary medium and in the absence of rotation and the magnetic field. In this case the solution reduces to

\[
u(\eta, t) = A \left( 1 - \frac{\cosh \frac{\sqrt{ARe} \eta}{2}}{\cosh \frac{\sqrt{ARe}}{2}} \right) e^{i\omega t},
\]

which is the well known solution reported by Schlichting and Gersten [26] for periodic variation of the pressure gradient along the axis of the channel.

From the velocity field we can now obtain the skin-friction \(\tau_L\) at the left plate in terms of its amplitude and phase angle as

\[
\tau_L = \frac{\partial u}{\partial \eta} = |F| \cos(t + \varphi),
\]

with \(F_r + i F_i = \frac{ARe}{m^2} \tanh \frac{m}{2t} + \frac{Gr}{(i\pi n^2 - m^2)} \left( \frac{m}{\sinh \frac{\pi}{2t}} - \frac{n}{\sinh \frac{\pi}{t}} \right) \). The amplitude is

\[
|F| = \sqrt{F_r^2 + F_i^2}
\]

and the phase angle \(\varphi = \tan^{-1} \frac{F_i}{F_r}\). From the temperature field given in equation (31) the heat transfer coefficient \(Nu\) (Nusselt number) in terms of its amplitude and the phase angle can be obtained as

\[
Nu = \frac{1}{\sinh(\pi/2)} = |H| \cos(\omega t + \psi),
\]

where \(H_r + i H_i = \frac{n}{\sinh(n)}\). The amplitude \(|H|\) and the phase angle \(\psi\) of the heat transfer coefficient \(Nu\) (Nusselt number) are given by

\[
|H| = \sqrt{H_r^2 + H_i^2}
\]

and \(\psi = \tan^{-1} \left( \frac{H_i}{H_r} \right)\) respectively. The temperature field and the amplitude and the phase of the Nusselt number need no further discussion because these have already been discussed in detail by Singh [27].

**DISCUSSION OF THE RESULTS**

The hydrodynamic mixed convection flow in an infinite vertical channel is analyzed when the entire system rotates about an axis perpendicular to the planes of the plates. In the presence of transverse magnetic field an exact solution of the problem is obtained. The velocity field and the shear stress in terms of its amplitude and phase angle are evaluated numerically for different sets of the values of rotation parameter \(\Omega\), viscoelastic parameter \(\gamma\), Reynolds number \(Re\), Hartmann number \(M\), Grashof number \(Gr\), Peclet number \(Pe\), radiation parameter \(N\), pressure gradient \(A\) and the frequency of oscillations \(\omega_0\). These numerical values are then shown graphically to assess the effect of each parameter for the two cases of small \((\Omega = 5)\) and large \((\Omega = 15)\) rotations.

Fig. 2 illustrates the variation of the velocity with the increasing rotation of the system. It is quite obvious from this figure that velocity goes on decreasing with increasing rotation \(\Omega\) of the entire system. The velocity profiles initially remain parabolic with maximum at the centre of the channel for small values of rotation parameter \(\Omega=5\) and then as rotation increases i.e., \(\Omega=10\), the velocity profiles flatten. For further increase in rotation \((\Omega= 15)\) the maximum of velocity profiles no longer occurs at the centre but shift towards the walls of the channel. It means that for large rotation there arise boundary layers on the walls of the channel. The effect of the viscoelastic parameter \(\gamma\) on the velocity profiles are shown in Fig. 3. The figure clearly shows that the velocity decreases tremendously with the increasing values of \(\gamma\) for the small \((\Omega = 5)\) and large \((\Omega= 15)\) rotation of the system. For given values of other parameters the velocity is maximum in the case of Newtonian fluid i.e., \(\gamma=0\).
The variations of the velocity profiles with the Grashof number $Gr$ are presented in Fig. 4. For small rotations ($\Omega=5$) the velocity increases with the increasing Grashof number. The maximum of the parabolic velocity profiles shifts toward right half of the channel due to the greater buoyancy force in this part of the channel due to the presence of hotter plate. For large rotation ($\Omega=15$) the Grashof number has opposite effect on the velocity profiles in the right half and the left half of the channel. In the right half there lies hot plate at $\eta = 1/2$ and heat is transferred from the hot plate to the fluid and consequently buoyancy force enhances the flow velocity further. In the left half of the channel the transfer of heat takes place from the fluid to the cooler plate at $\eta = -1/2$. Thus, the effect of Grashof number on the velocity is reversed i.e. velocity decreases with increasing $Gr$. The variations of the velocity profiles with the Reynolds number $Re$ are exhibited in Fig. 5. Two cases of small rotation ($\Omega=5$) and large rotation ($\Omega=15$) are considered to ascertain the effect of Reynolds number $Re$. For small $\Omega (=5)$ the velocity goes on increasing with increasing $Re$ and remains parabolic with maximum at the centerline. However, for large $\Omega (=15)$ although velocity increases with increasing $Re$ but the maximum of the velocity shift towards the walls of the channel.

The effects of the magnetic field on the velocity field are depicted in Fig. 6. It is observed that for both cases of small ($\Omega=5$) and large ($\Omega=15$) rotations velocity increase with increasing Hartmann number $M$. This means that the increasing Lorentz force due to increasing magnetic field strength resists the backward flow caused by the rotation of the system. Fig. 7 shows the variations of the velocity with the permeability of the porous medium $K$. It is observed from the figure that the velocity decrease with the increase of $K$ for both small ($\Omega=5$) and large ($\Omega=15$) rotations of the system. We find from Fig. 8 that with the increase of Peclet number $Pe$ the velocity decreases for small ($\Omega=5$) and large ($\Omega=15$) rotations both. The decrement in the velocity is more significant in the right half of the channel in which the hotter plate lies. Fig. 9 shows that the velocity decreases with increasing radiation parameter for small rotations ($\Omega=5$). However, for large rotation ($\Omega=15$), the velocity increases. From Fig. 10 it is evident that the velocity goes on increasing with the increasing favorable pressure gradient $A$ ($>0$). The velocity profiles for small rotation ($\Omega=5$) remain parabolic with maximum at the centerline. But for large rotation ($\Omega=15$) the maximum of the velocity shift towards the walls of the channel. The effect of the frequency of oscillations $\omega$ on the velocity is exhibited in Fig. 11. It is noticed that velocity decreases with increasing frequency $\omega$ for either case of channel rotation large or small.

The skin-friction $\tau_f$ in terms of its amplitude $|F|$ and phase angle $\varphi$ has been shown in Figs. 12 and 13 respectively. The effect of each of the parameter on $|F|$ and $\varphi$ is assessed by comparing each curve with dotted curves I in these figures. In Fig. 12 the comparison of the curves III, IV, VI and IX with dotted curve I indicate that the amplitude increases with the increase of Grashof number $Gr$, Reynolds number $Re$, permeability of the porous medium $K$ and the pressure gradient parameter $A$. Similarly the comparison of the curves II, V, VII, VIII and X with dotted curve I depicts that the skin-friction amplitude decreases with the increase of viscoelastic parameter $\gamma$, Hartmann number $M$, Peclet number $Pe$, radiation parameter $N$ and rotation parameter $\Omega$. It is obvious that $|F|$ goes on decreasing with increasing frequency of oscillations $\omega$. From Fig. 13 showing the variations of the phase angle of the skin-friction it is clear that there is always a phase lag because the values of $\varphi$ remain negative throughout. Comparing curves II, III, VI and X with dotted curve I it is observed that the phase lag increases with the increase of viscoelastic parameter $\gamma$, Grashof number $Gr$, permeability of the porous medium $K$ and rotation parameter $\Omega$. Also the comparison of curves IV, V, VII, VIII and IX with dotted curve I indicate that the phase lag decreases with the increase of Reynolds number $Re$, Hartmann number $M$, Peclet number $Pe$, radiation parameter $N$ and the pressure gradient $A$. Phase angle goes on increasing with increasing frequency of oscillations $\omega$.

REFERENCES

Figure 2. Velocity profiles for $\gamma=0.2$, $Gr=5$, $Re=0.5$, $M=2$, $K=0.2$, $Pe=1$, $N=1$, $A=5$, $\omega=5$ and $t=0$.

Figure 3. Velocity profiles for $Gr=5$, $Re=0.5$, $M=2$, $K=0.2$, $Pe=1$, $N=1$, $A=5$, $\omega=5$ and $t=0$. 
Figure 4. Velocity profiles for $\gamma=0.2$, $Re=0.5$, $M=2$, $K=0.2$, $Pe=1$, $N=1$, $A=5$, $\omega=5$ and $t=0$.

Figure 5. Velocity profiles for $\gamma=0.2$, $Gr=5$, $M=2$, $K=0.2$, $Pe=1$, $N=1$, $A=5$, $\omega=5$ and $t=0$. 
Figure 6. Velocity profiles for $\gamma=0.2$, $Gr=5$, $Re=0.5$, $K=0.2$, $Pe=1$, $N=1$, $A=5$, $M=5$ and $t=0$.

Figure 7. Velocity profiles for $\gamma=0.2$, $Gr=5$, $Re=0.5$, $M=2$, $Pe=1$, $N=1$, $A=5$, $\omega=5$ and $t=0$. 
Figure 8. Velocity profiles for $\gamma=0.2$, Gr=5, Re=0.5, M=2, K=0.2, N=1, A=5, $\omega=5$ and $t=0$.

Figure 9. Velocity profiles for $\gamma=0.2$, Gr=5, Re=0.5, M=2, K=0.2, Pe=1, A=5, $\omega=5$ and $t=0$. 
Figure 10. Velocity profiles for $\gamma=0.2$, $Gr=5$, $Re=0.5$, $M=2$, $K=0.2$, $Pe=1$, $N=1$, $\omega=5$ and $t=0$.

Figure 11. Velocity profiles for $\gamma=0.2$, $Gr=5$, $Re=0.5$, $M=2$, $K=0.2$, $Pe=1$, $N=1$, $A=5$ and $t=0$. 
Values of various parameters shown in Figures 12 and 13.

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Figure 12. Amplitude of the skin friction.

Figure 13. Phase of the skin friction.