ON CLOSURE OPERATOR IN NANO TOPOLOGICAL SPACE

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Abstract: The objective of this paper is to introduce Nwg-interior and Nwg-closure of a set in Nano topological spaces and investigate some of its topological properties. Also a comparative study has been made with other existing closure and interior operators with suitable examples. Further, various characteristics of Nwg-continuous function and Nwg-closed maps are discussed using Nwg-closure and Nwg-interior.

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1. INTRODUCTION


In this paper new operators namely Nwg-interior and Nwg-closure of a set is introduced and few of its properties are discussed. Throughout this paper (U,τ(X)) is a Nano Topological space with respect to X Where X ⊆ U , R is an equivalence relation on U, \( \frac{U}{R} \) denotes the family of equivalence classes of U by R. (V,τ_ρ(Y)) is a Nano Topological space with respect to Y Where Y ⊆ V , \( \frac{V}{R} \) is an equivalence relation on V, \( \frac{V}{R} \) denotes the family of equivalence classes of V by \( \frac{V}{R} \). (W,τ_ρ(Z)) is a Nano Topological space with respect to Z Where Z ⊆ W. \( \frac{W}{R} \) is an equivalence relation on W, \( \frac{W}{R} \) denotes the family of equivalence classes of W by \( \frac{W}{R} \).

II. PRELIMINARIES

This section is to recall some definitions and properties which are useful in this study.

**Definition:2.1[5]** Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X ⊆ U ,

1. The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is defined by \( L_{R}(X) = \bigcup_{x\in X} \{R(x): R(x) \subseteq X \} \). Where R(x) denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is defined by \( U_{R}(X) = \bigcup_{x\in X} \{R(x): R(x) \cap X \neq \phi \} \).
3. The boundary region of $X$ with respect to $R$ is the set of all objects, which can be classified neither as $X$ nor as not-$X$ with respect to $R$ and is defined by $B_R(X) = U_R(X) - L_R(X)$.  

**Definition: 2.2 [5]** Let $U$ be the universe, $R$ be an equivalence relation on $U$ and $\tau_R(X) = \{ U, \phi, L_R(X), U_R(X), B_R(X) \}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.

1. $U$ and $\phi \in \tau_R(X)$.
2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ forms a topology on $U$ called as the Nano topology on $U$ with respect to $X$. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets. Elements of $[\tau_R(X)]$ are called Nano closed sets.

**Definition: 2.3 [5]** If $(U, \tau_R(X))$ is a Nano Topological space with respect to $X$ where $X \subseteq U$ and if $A \subseteq U$ then the Nano interior of $A$ is defined as the union of all Nano open subsets of $A$ and it is denoted by $NInt(A)$. Nano interior is the largest Nano open subset of $A$.

**Definition: 2.4 [5]** The Nano closure of $A$ is defined as the intersection of all Nano closed sets containing $A$ and it is denoted by $Ncl(A)$. It is the smallest Nano closed set containing $A$.

**Definition: 2.5 [5]** Let $(U, \tau_R(X))$ be a Nano Topological space with respect to $X$ and $A \subseteq U$. Then $A$ is said to be

(i) Nano semi-open if $A \subseteq NCl(NInt(A))$  
(ii) Nano pre-open if $A \subseteq NInt(NCl(A))$  
(iii) Nano $\alpha$-open if $A \subseteq NInt(NCl(NInt(A)))$  
(iv) Nano Regular open if $A = NInt(NCl(A))$  

**Definition: 2.6** Let $(U, \tau_R(X))$ be a Nano Topological space. A subset $A$ of $(U, \tau_R(X))$ is called

(i) Nano generalized closed set [1] if $Ncl(A) \subseteq V$ where $A \subseteq V$ and $V$ is Nano open.  
(ii) Nano weakly generalized closed (briefly Nwg-closed) set [8] if $Ncl(NInt(A)) \subseteq V$ where $A \subseteq V$ and $V$ is Nano open.

**Definition 2.7[1]** If $(U, \tau_R(X))$ is a Nano topological space and $A \subseteq U$, the union of all Nano generalized open sets contained in $A$ is called the nano generalized Interior of $A$ and denoted by $NgInt(A)$. The intersection of all Nano generalized closed sets containing $A$ is called the nano generalized closure of $A$ and denoted by $Ngcl(A)$.

**Definition: 2.8 [12]** If $(U, \tau_R(X))$ is a Nano topological space and $A \subseteq U$, the union of all Nano regular open sets contained in $A$ is called the nano regular Interior of $A$ and denoted by $NrInt(A)$. The intersection of all Nano regular closed sets containing $A$ is called the nano regular closure of $A$ and denoted by $Nrcl(A)$.

**Definition: 2.9[9]** The map $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is Nwg-continuous on $U$ if the inverse image of every Nano closed (open) set in $V$ is Nano weakly generalized closed(open) in $U$.

**Definition: 2.10 [2]** The map $f : (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is Nwg-closed (open) map on $U$ if the image of every Nano-closed(open) set in $U$ is Nwg-closed(open) set in $V$.

**Theorem 2.11[9]**

(i) Every Nano open(closed) set is Nano weakly generalized open(closed) set.  
(ii) Every Nano generalized open(closed) set is Nano weakly generalized open(closed) set.  
(iii) Every Nano regular open(closed) set is Nano weakly generalized open(closed) set.

III. Nwg-INTERIOR & Nwg-CLOSURE
In this section we define the operators Nwg-interior and Nwg-closure in Nano Topological Space.

**Definition: 3.1** Let A be a subset a Nano topological space \((U, \tau_R(X))\). A point \(x \in A\) is said to be a Nano weakly generalized interior point of A if there exist a Nwg-open set G such that \(x \in G \subseteq A\).

The set of all Nano weakly generalized interior points of A is called Nano weakly generalized interior of A and is denoted by \(\text{NInt}_{\text{wg}}(A)\).

**Remark: 3.2** Nano weakly generalized interior of A is the union of all the Nwg-open set contained in A. \(\text{NInt}_{\text{wg}}(A) = \cup \{B : B \text{ is Nwg - open sets such that } B \subseteq A\}\).

**Example: 3.3** Let \(U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}\), \(X = \{2, 4, 6, 8, 9\}\), \(R = \{(x, y)/ x \text{ and } y \text{ are odd (or) even } x, y \in U\}\) \(U / R = \{[2, 4, 6, 8], [1, 3, 5, 7, 9]\}\) Then the Nano topology is \(\tau_R(X) = \{U, \phi, [2, 4, 6, 8], [1, 3, 5, 7, 9]\}\). Let \(A = \{1, 2, 4, 6, 8, 7\}\), all the Points of A are Nano weakly generalized interior of A, hence \(\text{NInt}_{\text{wg}}(A) = A\).

**Remark: 3.4** Let A and B the subsets of U then

(i) \(\text{NInt}_{\text{wg}}(U) = U\)   
(ii) \(\text{NInt}_{\text{wg}}(\phi) = \phi\)   
(iii) \(\text{NInt}_{\text{wg}}(A) \subseteq A\)   
(iv) \(\text{NInt}_{\text{wg}}(A)\) is a subset of A that can be written as the union of all Nwg-open sets   
(v) \(\text{NInt}_{\text{wg}}(A)\) need not be a Nwg-open set.

**Theorem: 3.5** For any \(A \subseteq (U, \tau_R(X))\), \(\text{NInt}(A) \subseteq \text{NInt}_{\text{wg}}(A)\)

**Proof:** Let \(x \in \text{NInt}(A)\), then there exist an Nano open set G such that \(x \in G \subseteq A\), Since every Nano open set is Nwg-open set , G is Nwg-open set containing x and \(G \subseteq A\). Hence x is an Nwg interior point. \(x \in \text{NInt}_{\text{wg}}(A)\). \(\text{NInt}(A) \subseteq \text{NInt}_{\text{wg}}(A)\).

**Theorem: 3.6** For any \(A \subseteq (U, \tau_R(X))\),

(i) \(\text{NgInt}(A) \subseteq \text{NInt}_{\text{wg}}(A)\),   
(ii) \(\text{NrInt}(A) \subseteq \text{NInt}_{\text{wg}}(A)\)

The converse of the theorem 3.5 and 3.6 need not be true as shown in the following examples.

**Example: 3.7** In example 3.3, let \(A = \{1, 2, 4, 6, 8, 7\}\), \(\text{NInt}(A) = \{2, 4, 6, 8\}\), \(\text{NInt}_{\text{wg}}(A) = \{1, 2, 4, 6, 8, 7\}\), \(\text{NInt}_{\text{wg}}(A) \not\subseteq \text{NInt}(A)\).

**Example: 3.8** Let \(A = \{a, b, c, d\}\), \(X = \{a, b, c\}\), \(U / R = \{\{a, b, c\}, \{d\}\}\) \(\tau_R(X) = \{U, \phi, \{a, b, c\}\}\) \(\text{NInt}_{\text{wg}}(A) = A\). \(\text{NgInt}(A) = \phi\) \(\text{NInt}_{\text{wg}}(A) \not\subseteq \text{NgInt}(A)\).

**Example: 3.9** Let \(A = \{a, b, c, d\}\) with \(U / R = \{\{a\}, \{c\}, \{b, d\}\}\) and \(X = \{a, b\}\). Then the Nano topology is \(\tau_R(X) = \{U, \phi, \{a\}, \{a, b\}, \{b, d\}\}\). Let \(A = \{b, c\}\), \(\text{NInt}_{\text{wg}}(A) = A\), \(\text{NrInt}(A) = \phi\) \(\text{NInt}_{\text{wg}}(A) \not\subseteq \text{NrInt}(A)\).

**Remark: 3.10** Let A be a subset a Nano Topological space \((U, \tau_R(X))\) and \(A\) is Nwg-open subset of a Nano topological space then \(\text{Nwg-Int}(A) = A\).

**Remark: 3.11** If \(\text{Nwg-Int}(A) = A\) then A need not be Nwg-open as shown in the following example.

**Example: 3.12** Let \(A = \{a, b, c, d\}\) with \(U / R = \{\{a\}, \{c\}, \{b, d\}\}\) and \(X = \{a, c\}\). Then the Nano topology is \(\tau_R(X) = \{U, \phi, \{a, c\}\}\). Let \(A = \{b, d\}\), \(\text{NInt}_{\text{wg}}(A) = A\) but it is not Nwg-open.

**Theorem: 3.13** Let A and B are the subset of U then

(i) If B is Nwg-open set contained in A then \(B \subseteq \text{NInt}_{\text{wg}}(A)\)   
(ii) If \(A \subseteq B\), then \(\text{NInt}_{\text{wg}}(A) \subseteq \text{NInt}_{\text{wg}}(B)\)
Proof

(i) Let $B$ is Nwg-open set contained in $A$. Let $x \in B$, then by definition $x$ is a Nwg-Interior point of $A$. Hence $x \in \text{NInt}_{wg}(A)$ which implies $B \subseteq \text{Nwg-Int}(A)$.

(ii) If $A \subseteq B$ and $x \in \text{NInt}_{wg}(A)$, there exist an Nwg-open set $G$ such that $x \in G \subseteq A$, since $A \subseteq B$, $x \in G \subseteq A \subseteq B$, Hence $x \in \text{NInt}_{wg}(B)$, $\text{NInt}_{wg}(A) \subseteq \text{NInt}_{wg}(B)$.

Definition: 3.14 Let $A$ be a subset a Nano topological space $U$. The intersection of all the Nwg-closed set containing $A$ is called Nwg-closure of $A$. 
\[ \text{NCl}_{wg}(A) = \bigcap \{ B : B \text{ is Nwg-closed set} \text{such that } A \subseteq B \} \]

Definition: 3.15 Let $A \subseteq U$. A point $x \in A$ is said to be a Nwg-limit point of $A$ if for each Nwg-open set $G$ containing $x$, $G \cap (A - \{x\}) \neq \emptyset$. The set of all Nwg-limit point of $A$ is called Nwg-derived set of $A$ and is denoted by $\text{ND}_{wg}(A)$.

Remark: 3.16 $A \cup \text{ND}_{wg}(A) = \text{NCl}_{wg}(A)$.

Remark: 3.17 Let $A$ is a subset of $U$ then

(i) $\text{NCl}_{wg}(U) = U$ (ii) $\text{NCl}_{wg}(\emptyset) = \emptyset$ (iii) $A \subseteq \text{NCl}_{wg}(A)$

(iv) $\text{NCl}_{wg}(A)$ is a set containing $A$ that can be written as the intersection of Nwg-closed sets.

(v) $\text{NCl}_{wg}(A)$ need not be Nwg-closed set.

Remark: 3.18 If $A$ is Nwg-closed subset of a Nano topological space then $\text{NCl}_{wg}(A) = A$.

Remark: 3.19 If $\text{NCl}_{wg}(A) = A$, then $A$ need not be Nwg-closed as seen from the following example.

Example: 3.20 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, c\}$. Then the Nano topology is $\tau_{B}(X) = \{U, \phi, \{a, c\}\}$. Let $A = \{a, c\}$, $\text{NCl}_{wg}(A) = A$ but $A$ is not Nwg-closed.

Theorem: 3.21 For subset $A$ of a Nano Topological space $(U, \tau_{B}(X))$,

(i) $\text{NInt}_{wg}(A) = A - \text{ND}_{wg}(U - A)$

(ii) $U - \text{NCl}_{wg}(A) = \text{NInt}_{wg}(U - A)$

(iii) $U - \text{NInt}_{wg}(A) = \text{NCl}_{wg}(U - A)$

Proof:

(i) Let $x \in \text{NInt}_{wg}(A)$, there is an Nwg-open set $G$ such that $x \in G \subseteq A$. $G \cap (U - A) - \{x\} = \phi$, $x \notin \text{ND}_{wg}(U - A)$. Therefore $x \in A$ and $x \notin \text{ND}_{wg}(U - A)$. Hence $x \in A - \text{ND}_{wg}(U - A)$ $\text{NInt}_{wg}(A) \subseteq A - \text{ND}_{wg}(U - A)$.

Conversely let $x \in A - \text{ND}_{wg}(U - A)$. $x \notin \text{ND}_{wg}(U - A)$ By definition there is an Nwg-open set such that $G \cap (U - A) - \{x\} = \phi$. Since $x \notin U - A$, $G \cap (U - A) = \phi$. Therefore $G \subseteq A$. Hence $G$ is an Nwg-open set containing $x$ such that $G \subseteq A$. Hence $x$ is a Nwg-interior point of $A$. Thus $x \in \text{NInt}_{wg}(A)$. $A - \text{ND}_{wg}(U - A) \subseteq \text{NInt}_{wg}(A)$.

(ii) $U - \text{NCl}_{wg}(A) = U - (A \cup \text{ND}_{wg}(A))$

$= (U - A) \cap (U - \text{ND}_{wg}(A))$

$= (U - A) - \text{ND}_{wg}(A)$
Since any Nano open set is \( Nwg - \{ wg \} \times X - \{ wg \} \) and \( B - \{ wg \} \subseteq A \), then the Nano topology is the intersection of \( A \) and \( B \). Therefore, \( x \in NCl(A) \). Hence \( NCl_wg(A) \subseteq NCl(A) \).

**Theorem 3.22** For \( A \subseteq (U, \tau_r(X)) \), then \( NCl_wg(A) \subseteq NCl(A) \).

**Proof:** Let \( x \in NCl_wg(A) \) and \( x \notin NCl(A) \). \( x \in U - Ncl(A) = Nnt(U - A) \), then there exist an Nano open set \( G \) containing \( x \) such that \( G 
subseteq U - A \). Since any Nano open set is \( Nwg \)-open set \( G \) is the \( Nwg \)-open set containing \( x \) such that \( G \subseteq U - A \). That \( x \in Nnt_wg(U - A) = U - Ncl_wg(A) \). Therefore, \( x \notin NCl_wg(A) \). There arises a contradiction. Therefore \( x \in NCl(A) \). Hence \( NCl_wg(A) \subseteq NCl(A) \).

**Example 3.24** Let \( U = \{a, b, c\} \), \( U / R = \{\{a, b\}, \{c\}\} \) and \( X = \{a, c\} \). Then the Nano topology is \( \tau_r(X) = \{U, \phi, \{c\}, \{a, b\}\} \). Let \( A = \{a\} \), \( NCl(A) = \{a, b\} \), \( NCl_wg(A) = A \), \( NCl(A) \nsubseteq NCl_wg(A) \).

**Example 3.25** Let \( U = \{a, b, c, d\} \), \( X = \{a, c\} \), \( U / R = \{\{a\}, \{b\}, \{c, d\}\} \). Then the Nano topology is \( \tau_r(X) = \{U, \phi, \{a\}, \{b\}, \{c, d\}\} \). Let \( A = \{c\} \), \( NgCl(A) = \{b, c\} \), \( NCl_wg(A) = A \), \( NgCl(A) \nsubseteq NCl_wg(A) \).

**Example 3.26** In the above example, \( A = \{d\} \), \( NrCl(A) = \{b, c, d\} \), \( NCl_wg(A) = A \), \( NrCl(A) \nsubseteq NCl_wg(A) \).

**Theorem 3.27** If \( A \) and \( B \) are subsets of \( U \), then \( (Nnt_wg(A)) \cup (Nnt_wg(B)) \subseteq Nnt_wg(A \cup B) \).

**Proof:** \( A \subseteq A \cup B \), \( B \subseteq A \cup B \)

\[ Nnt_wg(A) \subseteq Nnt_wg(A \cup B) \]

\[ Nnt_wg(B) \subseteq Nnt_wg(A \cup B) \]

\[ (Nnt_wg(A)) \cup (Nnt_wg(B)) \subseteq Nnt_wg(A \cup B) \]

**Theorem 3.28** If \( A \) and \( B \) are subsets of \( U \), then \( Nnt_wg(A \cap B) = Nnt_wg(A) \cap Nnt_wg(B) \).

**Proof:** \( U - Nnt_wg(A \cap B) = NCl_wg(U - (A \cap B)) \)

\[ Nnt_wg(A \cap B) = U - NCl_wg(U - (A \cap B)) \]

\[ = U - (NCl_wg(U - A) \cup NCl_wg(U - B)) \]

\[ = (U - NCl_wg(U - A)) \cap (U - NCl_wg(U - B)) \]

\[ Nnt_wg(A \cap B) = Nnt_wg(A) \cap Nnt_wg(B) \]
**Theorem: 3.29** Let $f : (U, \tau_u(X)) \rightarrow (V, \tau_v(Y))$ be a Nwg-continuous function then $f(NCl_{wg}(A)) \subseteq NCl(f(A))$ for every subset $A \subseteq U$.

**Proof:** Let $A \subseteq U$ then $NCl(f(A))$ is a Nano closed set in $V$. Since $f$ is Nwg-continuous $f^{-1}(NCl(f(A)))$ is Nwg-closed set in $U$ and $NCl_{wg}(f^{-1}(NCl(f(A)))) = f^{-1}(NCl(f(A)))$. $f(A) \subseteq NCl(f(A))$. $NCl_{wg}(A) \subseteq NCl_{wg}(f^{-1}(NCl(f(A))))$. $NCl_{wg}(A) \subseteq f^{-1}(NCl(f(A)))$. $f(NCl_{wg}(A)) \subseteq NCl(f(A))$.

**Remark: 3.30** The converse of the above theorem need not be true as shown in the following example.

**Example: 3.31** Let $U = \{a, b, c, d\}$ with $U / R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, d\}$. Then the Nano topology is $\tau_u(X) = \{U, \phi, \{a\}, \{c, d\}\}$.

Let $V = \{a, b, c, d\}$ with $V / R = \{\{a\}, \{b, c\}, \{d\}\}$ and $Y = \{b, c, d\}$.

Let $f : (U, \tau_u(X)) \rightarrow (V, \tau_v(Y))$ be a identity map from $(U, \tau_u(X))$ to $(V, \tau_v(Y))$. Then $f$ is not Nwg-continuous function because $f^{-1}([b, c, d]) = \{b, c, d\}$ is not Nwg-open in $U$. But $f(NCl_{wg}(A)) \subseteq NCl(f(A))$ for every subset $A \subseteq U$.

**Remark: 3.32** In theorem 3.29, if $f(A)$ is Nano closed set then $f(NCl_{wg}(A)) = NCl(f(A))$.

**Corollary: 3.33** Let $f : (U, \tau_u(X)) \rightarrow (V, \tau_v(Y))$ be a Nwg-continuous function then $f(NCl_{wg}(f^{-1}(A))) \subseteq (NCl(A))$ for every subset $A \subseteq V$.

**Theorem: 3.34** Let $f : (U, \tau_u(X)) \rightarrow (V, \tau_v(Y))$ be a Nwg-continuous function then $f^{-1}(NInt(A)) \subseteq NInt_{wg}(f^{-1}(A))$ for every subset $A \subseteq V$.

**Proof:** Let $A \subseteq V$ then $NIntA$ is a Nano open set in $V$. Since $f$ is Nwg-continuous $f^{-1}(NIntA)$ is Nwg-open set in $U$ and $NInt(f^{-1}(NIntA)) = f^{-1}(NIntA)$, $NIntA \subseteq A$, $f^{-1}(NIntA) \subseteq f^{-1}(A)$, $NInt_{wg}(f^{-1}(NIntA)) \subseteq NInt_{wg}(f^{-1}(A))$, $f^{-1}(NIntA) \subseteq NInt_{wg}(f^{-1}(A))$.

**Remark: 3.35** The converse of the above theorem need not be true as shown in the following example.

**Example: 3.36** In example 3.31, $f$ is not Nwg-continuous function, but $f^{-1}(NIntA) \subseteq NInt_{wg}(f^{-1}(A))$ for every subset $A \subseteq V$.

**Remark: 3.37** In theorem 3.34 $f^{-1}(NIntA) = NInt_{wg}(f^{-1}(A))$ if $A$ is Nano open.

**Theorem: 3.38** Let $f : (U, \tau_u(X)) \rightarrow (V, \tau_v(Y))$ be a Nwg-closed map then $NCl_{wg}(f(A)) \subseteq f(NCl(A))$ for every subset $A \subseteq U$.

**Proof:** Let $A \subseteq U$ then $NCl(A)$ is a Nano closed set in $U$. Since $f$ is Nwg-closed $f(NCl(A))$ is Nwg-closed set in $U$ and $NCl_{wg}(f(NCl(A))) = f(NCl(A))$. $A \subseteq NCl(A)$, $f(A) \subseteq f(NCl(A))$.

**Remark: 3.39** The converse of the above theorem need not be true as shown in the following example.
Example: 3.40 Let $U = \{a, b, c, d, e\}$ with $U \cap R = \{\{a, c\}, \{b\}, \{d\}, \{e\}\}$ and $X = \{a, b\}$. Then the Nano topology is $\tau_{N}(X) = \{U, \emptyset, \{b\}, \{a, c\}, \{a, b, c\}\}$.

Let $V = \{a, b, c, d, e\}$ with $V \cap R = \{\{a\}, \{b\}, \{c, d\}, \{e\}\}$ and $Y = \{a, b\}$. Then the Nano topology is $\tau_{N}(Y) = \{V, \emptyset, \{a, b\}\}$.

Let \( f : (U, \tau_{N}(X)) \rightarrow (V, \tau_{N}(Y)) \) be a function defined as \( f(a) = c, f(b) = d, f(c) = e, f(d) = a, f(e) = b \). \( f \) is not Nwg-closed function, but \( NCl_{wg}(f(A)) \subseteq f(NCl(A)) \) for every subset \( A \subseteq U \).

Remark: 3.41 In theorem 3.38 \( f^{-1}(NIntA) = NInt_{wg}(f^{-1}(A)) \) if \( A \) is Nano closed.

Corollary: 3.42 Let \( f : (U, \tau_{N}(X)) \rightarrow (V, \tau_{N}(Y)) \) be a Nwg-closed map then \( NCl_{wg}(A) \subseteq f(NCl(f^{-1}(A))) \) for every subset \( A \subseteq V \).

Theorem: 3.43 Let \( f : (U, \tau_{N}(X)) \rightarrow (V, \tau_{N}(Y)) \) be a Nwg-open map then \( f(NIntA) \subseteq NInt_{wg}(f(A)) \) for every subset \( A \subseteq U \).

Proof: Let \( A \subseteq U \) then \( NIntA \) is a Nano open set in \( U \). Since \( f \) is Nwg-open \( f(NIntA) \) is Nwg-open set in \( V \) and \( NInt_{wg}(f(NIntA)) = f(NIntA) \). \( NIntA \subseteq A \), \( f(NIntA) \subseteq f(A) \), \( NInt_{wg}(f(NIntA)) \subseteq NInt_{wg}(f(A)) \), \( f(NIntA) \subseteq NInt_{wg}(f(A)) \).

Remark: 3.45 The converse of the above theorem need not be true as shown in the following example

Example: 3.46 Let $U = \{a, b, c, d, e\}$ with $U \cap R = \{\{a, c\}, \{b\}, \{d\}, \{e\}\}$ and $X = \{a, b\}$. Then the Nano topology is $\tau_{N}(X) = \{U, \emptyset, \{b\}, \{a, c\}, \{a, b, c\}\}$.

Let $V = \{a, b, c, d, e\}$ with $V \cap R = \{\{a\}, \{b\}, \{c, d\}, \{e\}\}$ and $Y = \{a, b\}$. Then the Nano topology is $\tau_{N}(Y) = \{V, \emptyset, \{a, b\}\}$.

Let \( f : (U, \tau_{N}(X)) \rightarrow (V, \tau_{N}(Y)) \) be a function defined as \( f(a) = c, f(b) = d, f(c) = e, f(d) = a, f(e) = b \). The image of Nano open sets are \( f([b]) = [d], f([a, c]) = [c, e], f([a, b, c]) = [c, d, e] \). \( f \) is not Nwg-open map but \( f(NIntA) \subseteq NInt_{wg}(f(A)) \) for every subset \( A \subseteq U \).

Remark: 3.47 In theorem 3.43, \( f(NIntA) = NInt_{wg}(f(A)) \) if \( A \) is Nano open.

Corollary: 3.48 Let \( f : (U, \tau_{N}(X)) \rightarrow (V, \tau_{N}(Y)) \) be a Nwg-closed map then \( f(NIntf^{-1}(A)) \subseteq NInt_{wg}(A) \) for every subset \( A \subseteq V \).

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