ON THE SQUARE ve-DEGREE INDEX AND ITS POLYNOMIAL OF CERTAIN OXIDE NETWORKS

V.R. Kulli
Department of Mathematics
Gulbarga University, Gulbarga 585106, India
e-mail: vrkulli@gmail.com

Abstract: In this paper, we introduce the square ve-degree index of a molecular graph. Considering the square ve-degree index, we define the square ve-degree polynomial of a molecular graph. We determine the square ve-degree index and its polynomial of dominating oxide networks and regular triangulate oxide networks.

Keywords: square ve-degree index, dominating oxide network, regular triangulate oxide network.

Mathematics Subject Classification: 05C05, 05C07, 05C35.

1. INTRODUCTION:

Let $G$ be a finite, simple connected graph. Let $d_G(v)$ denote the degree of a vertex $v$ in $G$. The set of all vertices which adjacent to $v$ is called open neighborhood of $v$ and denoted by $N(v)$. The closed neighborhood set of $v$ is the set $N[v] = N(v) \cup \{v\}$. Let $S_v$ denote the sum of the degrees of all neighbors of a vertex $v$.

Numerous topological indices have found some applications in Theoretical Chemistry, see [1, 2]. Recently, Kulli [3] proposed the first hyper-ve-degree indices of a graph, defined as

$$HVe_1(G) = \sum_{uv \in E(G)} \left[d_{ve}(u) + d_{ve}(v)\right]^2.$$

For different ve-degree indices see [4, 5, 6].

Motivated by the definition of the first hyper ve-degree index, we introduce the square ve-degree index of a graph as follows:

The square ve-degree index of a graph $G$ is defined as

$$Q_{ve}(G) = \sum_{uv \in E(G)} \left[d_{ve}(u) - d_{ve}(v)\right]^2.$$ (1)

Considering the square ve-degree index, we introduce the square ve-degree polynomial of a graph $G$ as

$$Q_{ve}(G, x) = \sum_{uv \in E(G)} x^{d_{ve}(u) - d_{ve}(v)}.$$ (2)

For polynomials see [7, 8, 9, 10].

In this paper, the square ve-degree index and its polynomial of dominating oxide networks and regular triangulate oxide networks are computed. For oxide networks see [11].

2. RESULTS FOR DOMINATING OXIDE NETWORKS $DOX(n)$

In this section, we consider the graph of a dominating oxide network $DOX(n)$ see Figure 1.
Table 1. The partition of the edges with respect to their sum degree of end vertices of dominating oxide networks is given in

<table>
<thead>
<tr>
<th></th>
<th>(S_u, S_v)</th>
<th>(8, 12)</th>
<th>(8, 14)</th>
<th>(12, 12)</th>
<th>(12, 14)</th>
<th>(14, 16)</th>
<th>(16, 16)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>12n</td>
<td>12n–12</td>
<td>6</td>
<td>12n–12</td>
<td>24n–24</td>
<td>54n^2–114n+60</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. The ve-degree partition of the end vertices of edges for dominating oxide networks is given in

<table>
<thead>
<tr>
<th></th>
<th>(d_{ue}(u), d_{ve}(v))</th>
<th>(7, 10)</th>
<th>(7, 12)</th>
<th>(10, 10)</th>
<th>(10, 12)</th>
<th>(12, 14)</th>
<th>(14, 14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges</td>
<td>12n</td>
<td>12n–12</td>
<td>6</td>
<td>12n–12</td>
<td>24n–24</td>
<td>54n^2–114n+60</td>
<td></td>
</tr>
</tbody>
</table>

Theorem 1. The square ve-degree index of a dominating oxide network DOX(n) is

\[ Q_{ve}(DOX(n)) = 552n - 444. \]

Proof: Let G be the molecular graph of a dominating oxide network DOX(n). From equation (1) and Table 2, we deduce

\[
\begin{align*}
Q_{ve}(DOX(n)) &= \sum_{uv \in E(G)} \left( d_{ue}(u) - d_{ue}(v) \right)^2 \\
&= (7 - 10)^2 12n + (7 - 12)^2 (12n - 12) + (10 - 10)^2 6 + (10 - 12)^2 (12n - 12) \\
&+ (12 - 14)^2 (24n - 24) + (14 - 14) (54n^2 - 114n + 60) \\
&= 552n - 444.
\end{align*}
\]

Theorem 2. The square ve-degree polynomial of a dominating oxide network DOX(n) is

\[ Q_{ve}(DOX(n), x) = (12n - 12)x^{25} + 12nx^9 + (36n - 36)x^4 + (54n^2 - 114n + 66)x^0. \]

Proof: From equation (2) and Table 2, we deduce

\[
\begin{align*}
Q_{ve}(DOX(n), x) &= \sum_{uv \in E(G)} \left( d_{ue}(u) - d_{ue}(v) \right)^2 x^{2} \\
&= 12n x^{(7-10)^2} + (12n - 12) x^{(7-12)^2} + 6 x^{(10-10)^2} + (12n - 12) x^{(12-12)^2} \\
&+ (24n - 24) x^{(12-14)^2} + (54n^2 - 114n + 60) x^{(14-14)^2} \\
&= (12n - 12)x^{25} + 12nx^9 + (36n - 36)x^4 + (54n^2 - 114n + 66)x^0.
\end{align*}
\]

3. RESULTS FOR REGULAR TRIANGULATE OXIDE NETWORKS RTOX(n)

In this section, we consider a family of regular triangulate oxide networks which is denoted by RTOX(n), n≥3. The graph of RTOX(5) is shown in Figure 2.
The square ve-degree index of a regular triangulate oxide network $TROX(n)$ is

$$Q_{ve}(TROX(n)) = 174n - 88.$$  

**Proof:** From equation (1) and Table 4, we obtain

$$Q_{ve}(TROX(n)) = \sum_{uv \in E} \left( d_{ve}(u) \cdot d_{ve}(v) \right)^2$$

$$= (5-5)^2 + (5-10)^2 + 4 + (7-10)^2 + 4 + (7-12)^2 + (6n-8) + (10-10)^2 + (10-12)^2 + 6$$

$$+ (12-12)^2 + (6n-9) + (14-12)^2 + (6n-12) + (14-14)^2 + (3n^2 - 12n + 12)$$

$$= 174n - 88.$$  

**Theorem 4.** The square ve-degree polynomial of a regular triangulate oxide network $TROX(n)$ is

$$Q_{ve}(TROX(n), x) = (6n - 4)x^{25} + 4x^9 + (6n - 6)x^4 + (3n^2 - 6n + 6)x^0.$$  

**Proof:** From equation (2) and Table 4, we obtain

$$Q_{ve}(TROX(n), x) = \sum_{uv \in E} \left( d_{ve}(u) \cdot d_{ve}(v) \right)^2 x$$

$$= 2x^{(5-5)^2} + 4x^{(5-10)^2} + 4x^{(7-10)^2} + (6n-8)x^{(7-12)^2} + x^{(7-12)^2} + x^{(10-10)^2} + 6x^{(10-12)^2}$$

$$+ (6n-9)x^{(12-12)^2} + (6n-12)x^{(12-14)^2} + (3n^2 - 12n + 12)x^{(14-14)^2}.$$  

$$= (6n - 4)x^{25} + 4x^9 + (6n - 6)x^4 + (3n^2 - 6n + 6)x^0.$$  

**REFERENCES**


