ROUGH IDEALS AND THEIR PROPERTIES

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ABSTRACT. In this paper, we shall review the concept of rough ring, introduce the concept of rough ideal and give some properties of them. Also we define the concepts of rough ring homomorphism, rough ring anti-homomorphism in rough rings and study their properties.

Key words and phrases. rough ring, rough subring, rough ideal, rough ring homomorphism, rough ring anti-homomorphism.

1. INTRODUCTION

The theory of Rough set was proposed by Z Pawlak in 1982 [9]. Rough set theory, a new mathematical approach to deal with inexact, uncertain or vague knowledge, has recently received wide attention on the research areas in both of the real life applications and the theory itself. It is an extension of set theory in which a subset of universe is approximated by a pair of ordinary sets, called upper and lower approximations. A key concept in Pawlak rough set model is an equivalence relation, which are the building blocks for the upper and lower approximations. Combining the theory of rough set with abstract algebra is one of the trends in the theory of rough set. Some authors substituted an algebraic structure for the universal set and studied the roughness in algebraic structure. On the other hand, some authors studied the concept of rough algebraic structures. The concepts of rough group, rough semigroup and rough quotient group are studied in [8], [2] and [4]. B. Davvaz studied roughness in rings [3]. Su-Qing Han [10] proposed the concept of rough ring in an approximation space. He also proposed the concept of rough cosets and rough normal groups. The concept of rough ring, rough subring, rough ring homomorphism etc. are studied in [6]. In this paper, we shall review the concept of rough ring, introduce the concept of rough ideal. Also, we prove some properties about rough ring homomorphism and anti-homomorphism.

In section 2 we give the basic concepts of rough sets. Section 3 deals with the concepts of a rough ideal. In section 4 we define homomorphisms of rough rings and prove some related results. Section 5 discuss the properties of the anti-homomorphism in rough rings.

2. BASIC CONCEPTS OF ROUGH SETS

In this section we give the basic concepts of rough sets. For crisp algebraic concepts one may refer the books by Gallian [5] or Artin [1].

2.1. Definition. A pair \((U, \theta)\) where \(U \neq \emptyset\) and \(\theta\) is an equivalence relation on \(U\), is called an approximation space.

2.2. Definition. For an approximation space \((U, \theta)\) and a subset \(X\) of \(U\), the sets

\[
\overline{X} = \{x \in U / [x]_\theta \cap X \neq \emptyset\}
\]

\[
\underline{X} = \{x \in U / [x]_\theta \subseteq X\}
\]
are called upper approximation, lower approximation and boundary region of \( X \) in \((U, \theta)\), respectively.

2.3. **Proposition.** Let \( X, Y \subset U \). The following properties hold.

1. \( X \subset X \subset X \)
2. \( \emptyset = \emptyset = \emptyset, U = U = U \)
3. \( X \cap Y = X \cap Y \)
4. \( X \cap Y \subset X \cap Y \)
5. \( X \cup Y \subset X \cup Y \)
6. \( X \cup Y = X \cup Y \)
7. \( X \subset Y \) iff \( X \subset Y \) and \( X \subset Y \)

3. **Rough Ideal**

In this section we define and study about rough ideal.

3.1. **Definition.** Let \((U, \theta)\) be an approximation space and let \(+, *\) be two binary operations on \( U \). A subset \( R \) of \( U \) is called a rough ring if it satisfies the following properties.

I (1) \( \forall x, y \in R, \ x + y \in \overline{R} \).
(2) Associativity holds in \( \overline{R} \).
(3) \( \forall x \in R, \exists e \in \overline{R} \) such that \( x + e = e = x + e \), \( e \) is called additive rough identity.
(4) \( \forall x \in R, \exists y \in R \) such that \( x + y = e = y + x \), \( y \) is called additive rough inverse.
(5) \( \forall x, y \in R, x + y = y + x \).

(These first five conditions show that \( < R, + \) is a commutative additive rough group.)

II (1) \( \forall x, y \in R, \ x * y \in \overline{R} \).
(2) Associativity holds in \( \overline{R} \).

(These two conditions show that \( < R, * \) is a multiplicative rough semigroup.)

III (1) \( (x + y) * z = (x * z) + (y * z) \)
(2) \( x * (y + z) = (x * y) + (x * z) \ \forall x, y, z \in R. \)

(These conditions are the distributive property of multiplication over addition.)

3.2. **Definition.** A non-empty subset \( I \) of a rough ring \( R \) is said to be a rough right (left) ideal if

1. \( I \) is rough subgroup under addition
2. For every \( a \in I \) and \( r \in R, \ a * r \in \overline{I} \ (r * a \in \overline{I}) \).

If \( I \) is both rough right ideal and rough left ideal of \( R \), then it is called a rough ideal of \( R \). It is sometimes called a two-sided rough ideal.

**Remark.** A rough ideal is a rough subring.

3.3. **Definition.** A non-empty subset \( I \) of a rough ring \( R \) is said to be a rough bi-ideal if

1. \( I \) is rough subgroup under addition
2. For every \( a \in I \) and \( r \in R, \ a * r * a \in \overline{I} \).

**Example.** Let \( U = \{[0], [1], [2], ..., [8]\} \) be a set of equivalent classes with respect to modulo 9. Let \(+_9\) be the addition of equivalent classes and \(*_9\) be the multiplication of equivalent classes. A classification
of \( U \) is \( U/R = \{E_1, E_2, E_3\} \), where \( E_1 = \{[0], [1], [2]\} \), \( E_2 = \{[3], [4], [5]\} \), \( E_3 = \{[6], [7], [8]\} \). Let \( R = \{[0], [2], [7], [3], [6]\} \) and let \( I = \{[0], [3], [6]\} \). Then \( \overline{R} = E_1 \cup E_2 \cup E_3 = U \) and \( \overline{I} = E_1 \cup E_2 \cup E_3 = U \). Then \( \langle R, +, \ast, > \rangle \) is a rough ring. Now

(1) \( \forall x, y \in I, x + y \in I \).

(2) \( -[2] = [7] \in I, -[7] = [2] \in I, -[3] = [6] \in I, -[6] = [3] \in I, -[0] = [0] \).

(3) \( \forall x, y \in I, x \ast y \in I \).

(4) \( \forall x \in I \) and \( r \in R, x \ast r \in \overline{I} \) and \( r \ast x \in \overline{I} \).

Therefore \( I \) is a rough ideal of rough ring \( R \).

3.4. Theorem. Let \( I \) and \( J \) be two rough ideals of a rough ring \( R \). Then \( I \cap J \) is a rough ideal of \( R \) if \( \overline{I} \cap \overline{J} = T \cap \overline{J} \).

Proof. Suppose \( I \) and \( J \) are two rough ideals of rough ring \( R \). Then \( I \cap J \subseteq R \).

Consider \( x, y \in I \cap J \). This

\[ \Rightarrow x, y \in I, J \]
\[ \Rightarrow x + y \in \overline{I}, \overline{J} \] and \(-x \in I, J \) (\( \because I \) and \( J \) are rough subgroups)
\[ \Rightarrow x + y \in \overline{I} \cap \overline{J} \] and \(-x \in \overline{I} \cap \overline{J} \)

Assuming \( \overline{I} \cap \overline{J} = \overline{I} \cap \overline{J} \), we have \( x + y \in \overline{I} \cap \overline{J} \) and \(-x \in \overline{I} \cap \overline{J} \).

Thus \( I \cap J \) is a rough subgroup of \( R \).

Consider \( x \in I \cap J \) and \( r \in R \). This

\[ \Rightarrow r \ast x \in \overline{I} \] and \( r \ast x \in \overline{J} \) (\( \because I \) and \( J \) are rough ideals)
\[ \Rightarrow r \ast x \in \overline{I} \cap \overline{J} \]

Assuming \( \overline{I} \cap \overline{J} = \overline{I} \cap \overline{J} \), we have \( r \ast x \in \overline{I} \cap \overline{J} \). Also \( x \ast r \in \overline{I} \cap \overline{J} \). Therefore, \( I \cap J \) is a rough ideal of \( R \).

Remark. The union of two rough ideals need not be a rough ideals.

4. HOMOMORPHISM OF ROUGH RINGS

Let \((U_1, \theta_1)\) and \((U_2, \theta_2)\) be two approximation spaces and \(+, \ast, +', \ast'\) be binary operations over \(U_1\) and \(U_2\) respectively. Let \(R_1 \subseteq U_1\) and \(R_2 \subseteq U_2\) be two rough rings.

4.1. Definition. A mapping \(\phi : \overline{R}_1 \to \overline{R}_2\) satisfying

(1) \(\phi(x + y) = \phi(x) +' \phi(y)\)
(2) \(\phi(x \ast y) = \phi(x) \ast' \phi(y)\)

\(\forall x, y \in \overline{R}_1\), is called a rough ring homomorphism from \(R_1\) to \(R_2\).

4.2. Remark. In this case we simply say that \(\phi : R_1 \to R_2\) is a rough ring homomorphism, which means that the mapping \(\phi\) in fact from \(\overline{R}_1\) to \(\overline{R}_2\) and satisfies the above two conditions.

4.3. Definition. A rough ring homomorphism \(\phi : R_1 \to R_2\) is called

- a rough epimorphism (or surjective) if \(\phi : \overline{R}_1 \to \overline{R}_2\) is onto. That is \(\forall y \in \overline{R}_2, \exists x \in \overline{R}_1\) such that \(\phi(x) = y\)
• a rough embedding (or monomorphism) if \( \phi : R_1 \rightarrow R_2 \) is one-one.
• a rough isomorphism if \( \phi : R_1 \rightarrow R_2 \) is both one-one and onto.

4.4. Theorem. Let \( \phi : R_1 \rightarrow R_2 \) be a rough ring homomorphism and let \( I \) be a rough left (right) ideal of a rough ring \( R_1 \). Then \( \phi(I) \) is a rough left (right) ideal of rough ring \( R_2 \).

Proof. For \( x', y' \in \phi(I), \exists x, y \in I \) such that \( \phi(x) = x' \) and \( \phi(y) = y' \).

1. We have, \( \phi(x + y) = \phi(x) + ^* \phi(y) = x' + ^* y' \)
   Therefore, \( \forall \phi(x + y) \in \phi(I), \exists \phi(0) \in \phi(I) \)
   Since \( \phi(x + y) \in \phi(I) \), we get \( \phi(x + y) \in \phi(I) \).
   That is, \( x' + y' \in \phi(I) \)
(2) Since \( 0 \in I \), we get \( \phi(0) \in \phi(I) = \phi(I) \)
   Therefore \( \forall \phi(x) \in \phi(I), \exists \phi(0) \in \phi(I) \)
   Since \( \phi(x) + ^* \phi(0) = \phi(x + 0) = \phi(x) = \phi(0 + x) = \phi(0) + ^* \phi(x) \).
   Therefore, \( \phi(I) \) is a rough subgroup of \( R_2 \).
   Since \( I \) is a rough subgroup, for each \( x \in I \), additive rough inverse \( -x \in I \).
   Therefore, \( \phi(I) \) is rough subgroup of \( R_2 \).

4.5. Theorem. Let \( \phi : R_1 \rightarrow R_2 \) be a rough ring homomorphism and let \( I_2 \) be rough left (right) ideal of \( R_2 \). Then \( I_1 = \phi^{-1}(I_2) \) is a rough left (right) ideal of \( R_1 \) if \( \phi(I_1) = \phi(I_1) \) and \( \phi(R_1) = R_2 \).

Proof. Since \( I_1 = \phi^{-1}(I_2) \), we have \( \phi(I_1) = I_2 \), and so \( I_2 = \phi(I_1) = \phi(I_1) \)
   (1) \( \forall x, y \in I_1, \) we have \( \phi(x), \phi(y) \in I_2 \).
   Since \( I_2 \) is rough subgroup, we get \( \phi(x) + ^* \phi(y) \in I_2 \).
   That is, \( \phi(x + y) \in \phi(I_1) \).
   Thus \( x + y \in I_1 \).
   (2) \( \forall x \in I_1, \) we have \( \phi(x) \in I_2 \).
   Since \( I_2 \) is a rough subgroup, \( \phi(-x) = -\phi(x) \in I_2 \).
   That is, \( \phi(-x) \in \phi(I_1) \).
   Thus \( -x \in I_1 \).
   Therefore, \( I_1 = \phi^{-1}(I_2) \) is a rough subgroup of \( R_1 \).

4.6. Definition. Let \( R_1 \subseteq U_1 \) and \( R_2 \subseteq U_2 \) be two rough rings and \( \phi : R_1 \rightarrow R_2 \) be a rough ring homomorphism. Then \( \{ x/\phi(x) = 0, x \in R_1 \} \) where \( 0 \) is the additive rough identity of \( R_2 \), is called rough homomorphism kernel of \( \phi \), denoted by \( k\ker \phi \).

4.7. Theorem. Let \( \phi : R_1 \rightarrow R_2 \) be a rough ring homomorphism. Then rough homomorphism kernel is a rough ideal of \( R_1 \).

Proof. \( \forall x, y \in k\ker \phi, \phi(x) = 0, \phi(y) = 0 \).
   (1) We have \( \phi(x + y) = \phi(x) + ^* \phi(y) = 0 + ^* 0 = 0 \).
   Therefore, \( x + y \in k\ker \phi \)
(2) Since \( \phi(-x) = -\phi(x) = 0 \), we get \(-x \in \ker \phi \). Therefore, \( \ker \phi \) is a rough subgroup of \( R_1 \).

(3) We have \( \phi(x * r) = \phi(x) *' \phi(r) = 0 \). Therefore, \( x * r \in \ker \phi \). Similarly, \( r * x \in \ker \phi \).

Therefore \( \ker \phi \) is a rough ideal of \( R_1 \).

4.8. **Theorem.** Let \( \phi : \overline{R}_1 \rightarrow \overline{R}_2 \) be a rough ring epimorphism. Then \( \phi \) is a rough ring isomorphism if and only if rough ring homomorphism kernel is \( \{0\} \).

**Proof.** The necessary condition is obvious. We prove only the sufficient part.

Let \( a, b \in R_1 \) and \( \phi(a) = \phi(b) \). Then \( \phi(a - b) = \phi(a) - \phi(b) = 0 \). Therefore, \( a - b \in \ker \phi \).

Since \( \ker \phi = \{0\} \), \( a = b \). Therefore, \( \phi \) is a one-one mapping and also a rough ring isomorphism.

4.9. **Theorem.** Let \( \phi : \overline{R}_1 \rightarrow \overline{R}_2 \) be a rough ring homomorphism and let \( I \) be a rough bi-ideal of a rough ring \( R_1 \). Then \( \phi(I) \) is a rough bi-ideal of rough ring \( R_2 \) if \( \phi(\overline{I}) = \overline{\phi(I)} \) and \( \phi(R_1) = R_2 \).

**Proof.** By theorem (4.4), \( \phi(I) \) is rough subgroup of \( R_2 \).

\[ \forall \phi(r) \in R_2 \text{ and } \phi(x) \in \phi(I), \phi(x) *' \phi(r) *' \phi(x) = \phi(x * r * x) \]

Since \( I \) is a rough bi-ideal, we have \( x * r * x \in \overline{I} \).

\[ \Rightarrow \phi(x * r * x) \in \phi(\overline{I}) \]

\[ \Rightarrow \phi(x) *' \phi(r) *' \phi(x) \in \phi(\overline{I}) \]

\[ \Rightarrow \phi(x) *' \phi(r) *' \phi(x) \in \overline{\phi(I)} \]

\[ (\because \phi(\overline{I}) = \overline{\phi(I)}) \]

Therefore, \( \phi(I) \) is a rough bi-ideal of rough ring \( R_2 \).

4.10. **Theorem.** Let \( \phi : \overline{R}_1 \rightarrow \overline{R}_2 \) be a rough ring homomorphism and let \( I_2 \) be rough bi-ideal of \( R_2 \). Then \( I_1 = \phi^{-1}(I_2) \) is a rough bi-ideal of \( R_1 \) if \( \phi(\overline{I}_1) = \overline{\phi(I_1)} \) and \( \phi(R_1) = R_2 \).

**Proof.** Since \( I_1 = \phi^{-1}(I_2) \), we have \( \phi(I_1) = I_2 \), and so \( \overline{I}_2 = \overline{\phi(I_1)} = \phi(\overline{I}_1) \).

By theorem (4.5), \( I_1 = \phi^{-1}(I_2) \) is a rough subgroup of \( R_1 \).

\[ \forall r \in R_1 \text{ and } x \in I_1, \text{ we have } \phi(r) \in \phi(R_1) = R_2 \text{ and } \phi(x) \in I_2. \]

Since \( I_2 \) be rough bi-ideal of \( R_2 \), we have \( \phi(x) *' \phi(r) *' \phi(x) \in \overline{I}_2 \).

That is, \( \phi(x * r * x) \in \phi(\overline{I}_1) \). That is, \( x * r * x \in \overline{I}_1 \). Therefore, \( I_1 = \phi^{-1}(I_2) \) is a bi-rough ideal of \( R_1 \).

5. **ANTI-HOMOMORPHISM OF ROUGH RINGS**

Let \( (U_1, \theta_1) \) and \( (U_2, \theta_2) \) be two approximation spaces and \(+, *, +', *, ' \) be binary operations over \( U_1 \) and \( U_2 \) respectively. Let \( R_1 \subseteq U_1 \) and \( R_2 \subseteq U_2 \) be two rough rings.

5.1. **Definition.** A mapping \( \phi : \overline{R}_1 \rightarrow \overline{R}_2 \) satisfying

1. \( \phi(x + y) = \phi(x) +' \phi(y) \)
2. \( \phi(x * y) = \phi(y) *' \phi(x) \)

\( \forall x,y \in \overline{R}_1 \), is called a rough ring anti-homomorphism from \( R_1 \) to \( R_2 \).

5.2. **Theorem.** Let \( \phi : \overline{R}_1 \rightarrow \overline{R}_2 \) be a rough ring anti-homomorphism and let \( I \) be a rough right (left) ideal of a rough ring \( R_1 \). Then \( \phi(I) \) is a rough right (left) ideal of rough ring \( R_2 \) if \( \phi(\overline{I}) = \overline{\phi(I)} \) and \( \phi(R_1) = R_2 \).

**Proof.** For \( x', y' \in \phi(I) \), \( \exists x,y \in I \) such that \( \phi(x) = x' \) and \( \phi(y) = y' \).
(1) We have, \( \phi(x + y) = \phi(x) + \phi(y) = x' + y' \)
Since \( \phi(x + y) \in \phi(I) \), we get \( \phi(x + y) \in \phi(I) \). That is, \( x' + y' \in \phi(I) \)

(2) Since \( 0 \in I \), we get \( \phi(0) \in \phi(I) = \phi(\overline{I}) \)
Therefore \( \forall \, \phi(x) \in \phi(I) \), \( \exists \, \phi(0) \in \phi(I) \) such that
\( \phi(x) + \phi(0) = \phi(x + 0) = \phi(x) = \phi(0 + x) = \phi(0) + \phi(x) \).

(3) Since \( I \) is a rough subgroup, for each \( x \in I \), additive rough inverse \( -x \in I \).
Because \( -\phi(x) = \phi(-x) \in \phi(I) \), we get \( -\phi(x) \in \phi(I) \).
Therefore, \( \phi(I) \) is a rough subgroup of \( R_2 \).

(4) \( \forall \, \phi(r) \in R_2 \) and \( \phi(x) \in \phi(I) \), \( \phi(r) \ast' \phi(x) = \phi(x \ast r) \)
Since \( I \) is a rough right ideal, we have \( x \ast r \in \overline{I} \). Therefore, \( \phi(x \ast r) \in \phi(I) \). That is, \( \phi(r) \ast' \phi(x) \in \phi(I) \) \( \ast' \phi(x) \in \phi(\overline{I}) \).

Therefore, \( \phi(I) \) is a rough left ideal of rough ring \( R_2 \).
Similarly, we can prove the other statement.

5.3. Theorem. Let \( \phi : \overline{R_1} \rightarrow \overline{R_2} \) be a rough ring anti-homomorphism and let \( I_2 \) be rough left (right) ideal of \( R_2 \). Then \( I_1 = \phi^{-1}(I_2) \) is a rough right (left) ideal of \( R_1 \) if \( \phi(I_1) = \phi(\overline{I_1}) \) and \( \phi(R_1) = R_2 \).

Proof. Since \( I_1 = \phi^{-1}(I_2) \), we have \( \phi(I_1) = I_2 \), and so \( \overline{I_2} = \phi(\overline{I_1}) = \phi(I_1) \)
(1) \( \forall \, x, y \in I_1 \), we have \( \phi(x), \phi(y) \in I_2 \). Since \( I_2 \) is rough subgroup, we get \( \phi(x) + \phi(y) \in \overline{I_2} \).
That is \( \phi(x + y) \in \phi(I_1) \). Thus \( x + y \in \overline{I_1} \).

(2) \( \forall \, x \in I_1 \), we have \( \phi(x) \in I_2 \). Since \( I_2 \) is a rough subgroup, \( \phi(-x) = -\phi(x) \in I_2 \).
That is \( \phi(-x) \in \phi(I_1) \). Thus \( -x \in I_1 \).
Therefore, \( I_1 = \phi^{-1}(I_2) \) is a rough subgroup of \( R_1 \).

(3) \( \forall \, r \in R_1 \) and \( x \in I_1 \), we have \( \phi(r) \in \phi(R_1) = R_2 \) and \( \phi(x) \in I_2 \).
Since \( I_2 \) be rough left ideal of \( R_2 \), we have \( \phi(r) \ast' \phi(x) \in \overline{I_2} \). That is, \( \phi(x \ast r) \in \phi(I_1) \).
That is, \( x \ast r \in \overline{I_1} \).
Therefore, \( I_1 = \phi^{-1}(I_2) \) is a rough right ideal of \( R_1 \).
Similarly, we can prove the other statement also.

5.4. Definition. Let \( R_1 \subseteq U_1 \) and \( R_2 \subseteq U_2 \) be two rough rings and \( \phi : R_1 \rightarrow R_2 \) be a rough ring anti-homomorphism. Then \( \{x/\phi(x) = \overline{u}, x \in R_1 \} \) where \( \overline{u} \) is the additive rough identity of \( R_2 \), is called rough anti-homomorphism kernel of \( \phi \), denoted by \( ker \phi \).

5.5. Theorem. Let \( \phi : R_1 \rightarrow R_2 \) be a rough ring anti-homomorphism. Then rough anti-homomorphism kernel is a rough ideal of \( R_1 \).

Proof. \( \forall \, x, y \in ker \phi \), \( \phi(x) = \overline{u} \), \( \phi(y) = \overline{u} \).

(1) We have \( \phi(x + y) = \phi(x) + \phi(y) = \overline{u} + \overline{u} = \overline{u} \). Therefore, \( x + y \in ker \phi \)

(2) Since \( \phi(-x) = -\phi(x) = \overline{u}, \) we get \( -x \in ker \phi \). Therefore, \( ker \phi \) is a rough subgroup of \( R_1 \).

(3) We have \( \phi(x \ast r) = \phi(r) \ast' \phi(x) = \overline{u} \). Therefore, \( x \ast r \in ker \phi \). Similarly, \( r \ast x \in ker \phi \).

Therefore \( ker \phi \) is a rough ideal of \( R_1 \).

5.6. Theorem. Let \( \phi : R_1 \rightarrow R_2 \) be a rough ring anti-epimorphism. Then \( \phi \) is a rough ring anti-isomorphism if and only if rough ring anti-homomorphism kernel is \( \{0\} \).
5.7. **Theorem.** Let \( \phi : \overline{R_1} \rightarrow \overline{R_2} \) be a rough ring anti-homomorphism and let \( I \) be a rough bi-ideal of a rough ring \( R_1 \). Then \( \phi(I) \) is a rough bi-ideal of rough ring \( R_2 \) if \( \phi(I) = \overline{\phi(I)} \) and \( \phi(R_1) = R_2 \).

**Proof.** By theorem (5.2), \( \phi(I) \) is rough subgroup of \( R_2 \). \( \forall \phi(r) \in R_2 \) and \( \phi(x) \in \phi(I) \),
\[
\phi(x) \ast' \phi(r) \ast' \phi(x) = \phi(r \ast x) \ast' \phi(x) = \phi(x \ast r \ast x)
\]
Since \( I \) is a rough bi-ideal, we have \( x \ast r \ast x \in \overline{I} \).
\[
\Rightarrow \quad \phi(x \ast r \ast x) \in \phi(I)
\]
\[
\Rightarrow \quad \phi(x) \ast' \phi(r) \ast' \phi(x) \in \phi(I)
\]
\[
\Rightarrow \quad \phi(x) \ast' \phi(r) \ast' \phi(x) \in \overline{\phi(I)} \quad (\because \phi(I) = \overline{\phi(I)})
\]
Therefore, \( \phi(I) \) is a rough bi-ideal of rough ring \( R_2 \).

5.8. **Theorem.** Let \( \phi : \overline{R_1} \rightarrow \overline{R_2} \) be a rough ring anti-homomorphism and let \( I_2 \) be rough bi-ideal of \( R_2 \). Then \( I_1 = \phi^{-1}(I_2) \) is a rough bi-ideal of \( R_1 \) if \( \phi(I_1) = \overline{\phi(I_1)} \) and \( \phi(R_1) = R_2 \).

**Proof.** Since \( I_1 = \phi^{-1}(I_2) \), we have \( \phi(I_1) = I_2 \), and so \( \overline{I_2} = \overline{\phi(I_1)} = \phi(I_1) \)
By theorem (5.3), \( I_1 = \phi^{-1}(I_2) \) is a rough subgroup of \( R_1 \).
\( \forall r \in R_1 \) and \( x \in I_1 \), we have \( \phi(r) \in \phi(R_1) = R_2 \) and \( \phi(x) \in I_2 \).
Since \( I_2 \) be rough bi-ideal of \( R_2 \), we have \( \phi(x) \ast' \phi(r) \ast' \phi(x) \in \overline{I_2} \). That is, \( \phi(x \ast r \ast x) \in \phi(I_1) \). That is, \( x \ast r \ast x \in \overline{I_1} \). Therefore, \( I_1 = \phi^{-1}(I_2) \) is a rough bi-ideal of \( R_1 \).

6. **CONCLUSION**

In this paper, we have shown that the theory of rough sets can be extended to ideals in rings. We discussed the concept of rough ring homomorphism and rough ring anti-homomorphism. Also, we discussed homomorphic and anti-homomorphic properties of rough ideals. In a similar fashion the theory of rough sets can be extended to other topics in ring theory.

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