USING PARAMETRIC STATISTICAL PERFORMANCE MEASURE FOR QUALITY ASSESSMENT OF MULTIOBJECTIVE MULTISTAGE ANT COLONY OPTIMIZATION SYSTEM

A. A. Mousa
Department of Basic Engineering Science, Faculty Of Engineering, Shebin El-Kom, Minoufia University, Egypt, E_mail: a_mousa15@yahoo.com

Department of Mathematics, faculty of sciences, Taif University, Saudi Arabia

Abstract: This paper is focused on using parametric statistical test of the results in the field of population-based techniques. It proposes a Parametric Statistical test as a performance measure for quality assessment for Multiobjective Multistage Ant Colony Optimization MM-ACO system that optimize a fuzzy multi-objective resource allocation problem (FM-RAP), where the experimental analysis on the performance of a proposed algorithm is a crucial and necessary task to carry out in this research. Our approach has two characteristic features. Firstly, a set of nondominated solutions is obtained by exploring the optimal Pareto frontier using different $\alpha$ cut level and subsequently, based on the parametric statistical test, performance measure for the quality assessment was implemented in order to establish the viability of our algorithm, and to compare the results obtained by running our algorithms with the results obtained from applying particle swarm optimization. Furthermore, we provided an example of optimum utilization of human resources in reclamation of derelict land.

1. INTRODUCTION

Recently, there has been a boom in applying evolutionary algorithms to solve multiobjective optimization problems [1-5]. Evolutionary algorithms (EAs) are stochastic search methods that mimic the metaphor of natural biological evolution and/or the social behavior of species. The development of meta-heuristic optimization methods has been flourishing. Many meta-heuristic algorithms such as genetic algorithm [6], simulated annealing [7], tabu search [8], and swarm intelligence [9,10] have shown their efficacy in solving computationally intensive problems. Recently ant colony algorithm [11-15] has become an interesting approach to solve many hard problems because it combines and extends the attractive features. and has encouraged many researchers to develop ACO variants for tackling well-known NP-hard problems, such as the traveling salesman problem [16], quadratic assignment problem [17], scheduling problem [18], minimum weight vertex cover problem and curve segmentation problem [19], just to name a few.

This paper is formally, a natural extension of the work already given by the author Mousa and El-Desoky 2013[5] based on developing new hybrid ant colony system to detecting stable Pareto optimal solution, but the present approach introduces Parametric Statistical test as a performance Measure for Quality Assessment for the proposed algorithm. To confirm this analysis we perform a test to analyze if there is a statistically significant difference between the performances of the hybrid ACO algorithm [5] and swarm optimization algorithm [4]. Furthermore, an example of optimum utilization of human resources in reclamation of derelict land under fuzziness was provided. Since there is instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices, for this reasons a fuzzy representation of the multiobjective human resource allocation FM-RAP has been defined, where the input data involve many parameters whose possible values may be assigned by the experts. Based on $\alpha$ level concept [5], FM-RAP can be transformed to multiobjective resource allocation problem (M-RAP) at certain degree of $\alpha$.

2. MATHEMATICAL FORMULATION

In Multiobjective optimization (MO), a set of nondominated solutions is usually produced instead of single recommended solution. According to the concept of nondominance, a solution to a MO problem is nondominated, or Pareto optimal, if no objective can be improved without worsening at least one other objective. Naturally, these objective functions and constraints involve many parameters whose possible values may be assigned by the experts. In the traditional approaches, such parameters are fixed at some values in an experimental and/or subjective manner through the experts' understanding of the nature of the parameters. In practice, however, it is natural to consider that the possible values of these parameters are often only ambiguously known to experts' understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers. A fuzzy multiobjective resource allocation problem FM-RAP is defined as follows:
FM-RAP

\[
\begin{align*}
\text{Max } & z_1(y) = \sum_{i=1}^{N} f_1(y_i, \tilde{a}_i) \\
\text{Max } & z_2(y) = \sum_{i=1}^{N} f_2(y_i, \tilde{a}_i) \\
\vdots & \quad \vdots \quad \vdots \\
\text{Max } & z_k(y) = \sum_{i=1}^{N} f_k(y_i, \tilde{a}_i) \\
\text{S.t.} & \quad G_0(y) = \sum_{i=1}^{N} g_i(y_i, \tilde{a}_i) \leq M \\
& \quad y_i = 0, 1, \ldots, M \quad \forall i
\end{align*}
\]

(1)

Assuming that all \( K \) objectives should be maximized, without loss of generality. According to the above-mentioned mathematical model, assign \( M \) staff to \( N \) different activities for maximizing the benefit and minimizing the cost, etc. subject to one resource constraint. It can be formulated in a multiobjective integer programming model as follows:

\[
\begin{align*}
\text{Max } & z_1(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} \tilde{a}_{ij}^1 x_{ij} \\
\text{Max } & z_2(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} \tilde{a}_{ij}^2 x_{ij} \\
\vdots & \quad \vdots \quad \vdots \\
\text{Max } & z_k(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} \tilde{a}_{ij}^k x_{ij} \\
\text{S.t.} & \quad G_0(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} jx_{ij} \leq M, \quad G_i(x) = \sum_{j=1}^{M} x_{ij} = 1, \forall i, i
\end{align*}
\]

(2)

Notations
Indices
\( i \) Index of stage (activity) , \( i = 1, 2, \ldots, N \),
\( j \) Index of staff, \( j = 1, 2, \ldots, M \).
Parameters
\( N \) Total number of stages
\( M \) Total number of staff
\( \tilde{a}_{ij} \) fuzzy parameter represent the effectiveness of resource \( i \) when allocated to activity \( j \)

Decision variables
\[
x_{ij} = \begin{cases} 
1, & \text{if staff index } j \text{ is assigned to stage } i \\
0, & \text{otherwise.}
\end{cases}
\]
Fuzzy parameters are assumed to be characterized as the fuzzy numbers. The real fuzzy numbers \( \tilde{a} \) form a convex continuous fuzzy subset of the real line whose membership function \( \mu_{\alpha}(a) \) is defined by:

1) a continuous mapping from \( \mathbb{R}^1 \) to the closed interval \([0,1]\);
2) \( \mu_{\alpha}(a) = 0 \) for all \( a \in (-\infty, a_1] \);
3) strictly increasing on \([a_1, a_2] \);
4) \( \mu_{\alpha}(a) = 1 \) for all \( a \in [a_2, a_3] \);
5) strictly decreasing on \([a_3, a_4] \);
6) \( \mu_{\alpha}(a) = 0 \) for all \( a \in [a_4, +\infty) \);

Assume that \( \tilde{a} \) in the FM-RAP are fuzzy numbers whose membership functions are \( \mu_{\alpha}(a) \).

**Definition 1.** (\( \alpha \)-level set). The \( \alpha \)-level set or \( \alpha \)-cut of the fuzzy numbers \( \tilde{a} \) is defined as the ordinary set \( L_{\alpha}(\tilde{a}) \) for which the degree of their membership functions exceeds the level \( \alpha \in [0,1] \):

\[
L_{\alpha}(\tilde{a}) = \{ a \mid \mu_{\alpha}(a) \geq \alpha \}.
\]

(3)

For a certain degree \( \alpha \), the (FM-RAP) can be represented as a nonfuzzy \( \alpha \)M-RAP as follows:

\[
\begin{align*}
\text{Max} & \quad z_1(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} a_i^1 x_{ij} \\
\text{Max} & \quad z_2(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} a_i^2 x_{ij} \\
\text{Max} & \quad z_k(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} a_i^k x_{ij} \\
\text{S.t.} & \quad G_0(x) = \sum_{i=1}^{N} \sum_{j=1}^{M} j x_{ij} \leq M \\
& \quad G_i(x) = \sum_{j=1}^{M} x_{ij} = 1, \forall i, \\
& \quad L_{a_{ij}} \leq a_i^* \leq U_{a_{ij}} \\
& \quad x_{ij} = 0 \text{ or } 1, \forall i, j.
\end{align*}
\]

(4)

**Definition 2.** (\( \alpha \)-Pareto optimal solution). \( x^* \in X \) is said to be an \( \alpha \)-Pareto optimal solution to the (\( \alpha \)M-RAP), if and only if there does not exist another \( x \in X \), \( a \in L_{\alpha}(\tilde{a}) \) such that \( f_i(x, a_i) \geq f_i(x^*, a_i^*) \), \( i = 1,2,\ldots,k \), with strictly inequality holding for at least one \( i \), where the corresponding values of parameters \( a_i^* \) are called \( \alpha \)-level optimal parameters.

3. The Network Model of M-RAP

The decision-makers want to determine the optimal multiobjective human resource allocation path. Each node denotes two objectives of minimum cost and maximum profit. Where \( i = 1,2,\ldots,N \) is the index of activity stage, \( j = 1,2,\ldots,M \) is the index of staff (source), \( P_{ij} \) is the Expected profit of stage \( i \) when staff index \( j \) is assigned to it and \( C_{ij} \) is Expected cost of stage \( i \) when staff index \( j \) is assigned to it. In order to be able to tackle the multiobjective problems, initially, the problem to
be solved is represented by a connected graph, each stage represents one activity, each vertex represents index of staff and each edge represents a connection between two stages. At this point, M-RAP are represented as a network model, where limited supply represented by process activity (stage) \( i = (1, 2, \ldots, N) \) and limited resources represented by index of staff \( j = (1, 2, \ldots, M) \) as shown in figure 1.

![Diagram showing a network model representing MHRAP](image)

This model represents a special case of a combinatorial problem that is suitable for ant colony optimization algorithms. ACO searches the optimal solution by iteratively distributing a population of artificial ant agents (which shall be referred to as ant, for simplicity). At each iteration, a number of artificial ants are considered. Each of ants builds a solution by walking from stage to another on the graph with the constraint of ensures that we ensure that for each stage \( i \) we can only assign staff for it one time. At each step of the solution procedure, an ant selects the vertex from each stage to be visited according to a stochastic mechanism that is biased by the pheromone information. At vertex \( i \), the following vertex in the next stage is selected stochastically among alternatives. At the end of iteration, on the basis of the quality of the solutions constructed by the ants, the pheromone values are modified in order to bias ants in the next iterations to construct different solutions. Before entering the next iteration, the quantity of pheromone on each vertex is updated by the pheromone updating rule which states that the vertex along the paths that correspond to high quality candidate solutions will receive more quantities of pheromone during the current iteration. The ACO will report the global best solution observed when the stopping criterion (e.g. maximal number of iterations) is satisfied.

4. M-RAP via multistage decision based multipheromone ACO

In the proposed MM-ACO, a colony of agents is assigned for each objective. All the colonies have the same number of ants. All the ants in one colony try to find a solution at the same time according to heuristic information. Solutions found for all colonies in one cycle are evaluated according to the objective functions [5]. After completion of a cycle, the global pheromone trails are updated. The main steps of the proposed algorithm will be discussed in details as follows:

**Step 0. (Construct K Colonies):** In a multiobjective optimization, multiobjective functions \( F = (f_1, f_2, \ldots, f_k) \), need to be optimized simultaneously, there does not necessarily existence a solution that is best with respect to all objectives because of incommensurability and confliction among objectives. For this step, the number of colonies is set to \( K \) with its own pheromone structure, where \( K = |F| \) is the number of objectives to optimize.

**Step 1. (Initialization):** The problem of human resources allocation considers determining a vector of allocation path in the M-states (staff) and the N stages (different activity) under the minimum costs and the maximum profits. Each ant is composed of one vector path to allocate M staff to N stages.

In order to form the appropriate design of agent using MM-ACO, consider a problem of allocating 10 staff index to 4 activity. Firstly, consider a random selection element from 1 to 10 in each stage to form the available alternative nodes in each stage. Figure 2 shows the staff allocation path under the four-stage situation, where the line denotes the allocation path. Secondly, Figure 3 shows the structure of the staff for four-stage allocation path, node 3 was selected in stage 2 via measures (heuristic information, pheromone information) associated with each objectives, so as to select the successive nodes according to visibility and pheromone quantity, as similar node 3 was selected in stage 3. Finally, node 2 was selected in stage 4.
First, pheromones trails are initialized to a given value $\tau_0$ where $\tau_0$ is the pheromone information in the current iteration and Pareto set are initialized to an empty set. In fact, the solution of the problem is to find a path between two nodes $(x_{1j}, j \in (1, 2, ..., M))$ (source node) and $x_{Nj}$ (terminal node) having e.g., the maximum profits and minimum costs. Thus, the structure of each individual can express a path, i.e., a path is represented as a sequence of nodes.

**Step 2. (Solution Construction)** The goal is to minimize the total cost and maximize the total profits, etc. of the allocation path, which contains each stage exactly once. Heuristic information $\eta^k$ is defined for every objective function $f^k \in F$ such that for objective function that minimizes the cost, heuristic information $\eta^c = \frac{1}{\text{Price of Stuff}_c}$, and for objective function that maximizes the profit, heuristic information $\eta^p = \frac{1}{\text{Profit for Staff}_C}$, where each colony considers a single different objective, using its own pheromone structure and heuristic information to build its own solution. The task of each ant consists in the construction of a feasible M-RAP solution. Each ant constructs a solution as follows. First, one of the nodes of the M-RAP graph is randomly chosen as start node in the first stage. Then, the ant builds a tour in the M-RAP graph by moving in each construction step from its current node (i.e., the index of staff in which it is located) to another node which is under construction in the next stage. At each stage the traversed node is added to the solution under construction. When no nodes are left the ant closes the tour. This way of constructing a solution implies that an ant has a memory to store the already visited nodes. Each solution construction stage is performed as follows. Assuming the ant to be in node $i$ in the stage 1, the assignment of $j$ node to the next stage is done with probability:

$$P^U_j(x_j) = \frac{[\tau^U_j]^\sigma [\eta^U_j]^\beta}{\sum_{l \in K_U} [\tau^U_l]^\sigma [\eta^U_l]^\beta}, \quad \forall j \in M, \quad l \in K_U \quad (5)$$
Where, \( U_M \) is the set of candidate nodes in \( M \), which can be chosen by the \( U \)th ant, and removing nodes that violate constraints. \( \tau^K_j \) and \( \eta^K_j \) respectively are the pheromone and the heuristic factors of the candidate node \( j \) to select in the successive stage, and \( \sigma \) and \( \beta \) are two parameters that determine their relative importance pheromone trail and the heuristic value (note that \( \sigma \) was used instead of \( \alpha \) which is commonly used in ACO algorithm).

**Step 3. (Evaluation of Nondominated Solutions):** There usually exist a set of solutions for the multiobjective case which cannot simply be compared with each solutions, where no improvement in any objective function is possible without sacrificing at least one of the other objective functions \([5]\). Using this concept, all nondominated solutions are stored in the archive.

**Step 4. (Update Pheromone Trails):** When updating pheromone trails, one has to decide on which of the constructed solutions laying pheromone. A first possibility is to reward solutions that find the best values for each criterion in the current cycle, as proposed in \([20]\). A second possibility is to reward every non-dominated solution of the current cycle. In this case, one may either reward all the solutions in the Pareto set, as proposed in \([21]\) or only the new nondominated solution that enter in the set in the current cycle. The quantity of pheromone laying on a component represents the past experience of the colony with respect to choosing this component. Then, at each cycle every ant constructs a solution, and pheromone trails are updated. Once all ants have constructed their solutions, pheromone trails are updated as usually in ACO algorithms: first, pheromone trails are reduced by a constant factor to simulate evaporation to prevent premature convergence; then, some pheromone is laid on components of the best solution. Before activating the next iteration, the quantity of pheromone on each node is updated as follows:

\[
\tau^\ast = (1 - \rho) \tau^\ast + \sum_{u=1}^{U} \Delta \tau^u
\]

Where \( \rho \) is the evaporation rate of pheromone trails, such that \( 0 \leq \rho \leq 1 \) and \( mb \) is the size of elite ants \( \Delta \tau^u \) is the quantity of pheromone laid on the path by the \( t \)th ant during the current iteration, then

\[
\Delta \tau^u = \begin{cases} 
Q & \text{if ant } u \text{ used the path } e \text{ in its tour} \\
0 & \text{otherwise}
\end{cases}
\]

Where Q is a small constant and \( f_u \) denotes the objective value of the MOHRAP for a path of \( u \)th ant.

**Step 5. (Archive Update):** During the optimization process, the archive is updated. where, the current nondominated solutions are compared to those in the archive; the dominated ones are removed and the nondominated solutions are added to the archive. The size of this set needs to be kept reasonable, which may imply to sometimes remove nondominated solutions. The solutions belonging to the most populated areas can be removed first. Without loss of generality, the elements discussed above are synthesized to evolve the proposed MM-ACO algorithm. The pseudo code of the proposed algorithm is given in Figure 4.
for objective, k=1:K do
\[ f_k = f(X) \]
end for
Pareto \( \leftrightharpoons \) Pareto (X)
Archive \( \leftrightharpoons \) Archive \( \cup \) Pareto
Evaporation (Pheromone Information, \( \rho \))
Update trail (Different strategies are used (Pareto))
end while
Output \( \leftrightharpoons \) Archive elements

Fig. 4. The pseudo code of the proposed algorithm

5. Experimental results

This section is devoted to the discussion of the experimental results. We consider the application of reclamation of derelict land, where four stages was considered, which affect the optimization of the agriculture reclamation, including land settlement, land planning, digging of canals and plant cultivation[5,22]. Also, two objectives: maximizing benefit of reclamation and minimizing the costs of agriculture reclamation. The annually data \( \{a_{ij}^k, i = 1,2,...,10; j = 1,2,...,4; k = 1,2\} \) of the ten staff are given in table 1, where for each staff; we provide the minimum cost and maximum profit for the four stage land reclamation.

<table>
<thead>
<tr>
<th>Staff index</th>
<th>Land settlement</th>
<th>Land planning</th>
<th>Digging of canals</th>
<th>Plant cultivation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cost ( a_{11} )</td>
<td>Profit ( a_{12} )</td>
<td>Cost ( a_{21} )</td>
<td>Profit ( a_{22} )</td>
</tr>
<tr>
<td>1</td>
<td>790.0</td>
<td>138.0</td>
<td>770.0</td>
<td>147.0</td>
</tr>
<tr>
<td>2</td>
<td>800.0</td>
<td>129.0</td>
<td>710.0</td>
<td>141.0</td>
</tr>
<tr>
<td>3</td>
<td>720.0</td>
<td>121.0</td>
<td>630.0</td>
<td>134.0</td>
</tr>
<tr>
<td>4</td>
<td>680.0</td>
<td>106.0</td>
<td>660.0</td>
<td>127.0</td>
</tr>
<tr>
<td>5</td>
<td>610.0</td>
<td>114.0</td>
<td>550.0</td>
<td>121.0</td>
</tr>
<tr>
<td>6</td>
<td>530.0</td>
<td>97.0</td>
<td>510.0</td>
<td>113.0</td>
</tr>
<tr>
<td>7</td>
<td>470.0</td>
<td>91.0</td>
<td>470.0</td>
<td>101.0</td>
</tr>
<tr>
<td>8</td>
<td>350.0</td>
<td>83.0</td>
<td>410.0</td>
<td>97.0</td>
</tr>
<tr>
<td>9</td>
<td>280.0</td>
<td>65.0</td>
<td>350.0</td>
<td>91.0</td>
</tr>
<tr>
<td>10</td>
<td>310.0</td>
<td>66.0</td>
<td>310.0</td>
<td>85.0</td>
</tr>
</tbody>
</table>

Table 1 The average profit and cost of staff that we need for reclamation
Naturally, these data (The average profit and cost) involve many controlled parameters whose possible values are vague and uncertain. Consequently each numerical value in the domain can be assigned a specific "grade of membership" where 0 represents the smallest possible grade of membership, and 1 is the largest possible grade of membership. Thus fuzzy parameters can be represented by its membership grade ranging between 0 and 1.

The fuzzy numbers shown in figure 5 have been obtained from interviewing decision maker DM or from observing the instabilities in the global market and rate of prices fluctuations [5]. The idea is to transform a problem with these fuzzy parameters to a crisp version using $\alpha$ -cut level. This membership function can rewrite as follows:

$$
\mu(a_{ij}) = \begin{cases} 
1, & a = a_{ij} \\
\frac{20a}{a_{ij}} - 19 & 0.95a_{ij} \leq a \leq a_{ij} \\
21 - \frac{20a}{a_{ij}} & a_{ij} \leq a \leq 1.05a_{ij} \\
0 & a < 0.95a_{ij} \text{ or } a > 1.05a_{ij}
\end{cases} \quad (15)
$$

So, every fuzzy parameter $\tilde{a}_{ij}$ can be represented using the membership function in equation 15. By using $\alpha$ -cut level, these fuzzy parameters can be transformed to a crisp one having upper and lower bounds $[a_{ij}^L, a_{ij}^U]$, which declared in figure 5. Consequently, each $\alpha$ -cut level can be represented by the two end points of the alpha level. For more details the reader is referred to the reference Mousa and El-Desoky 2013[5].

The techniques used in this study were developed and implemented on dual 1.8 GHz, Core 2, PC using MATLAB environment. Our algorithm employs a set of parameters. The experiment environment and the setting values are depicted in table 2.
Table 2. The parameter setting

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of objective functions</td>
<td>2</td>
</tr>
<tr>
<td>Number of ants for colony 1</td>
<td>50</td>
</tr>
<tr>
<td>Number of ants for colony 2</td>
<td>50</td>
</tr>
<tr>
<td>Number of iteration</td>
<td>5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2</td>
</tr>
<tr>
<td>Q</td>
<td>100</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>10</td>
</tr>
</tbody>
</table>

Here, the problem is how to determine the optimal multiobjective human resource allocation path for considering the minimum cost and the maximum profit objectives simultaneously [22]. In order to efficiently and effectively obtain the solution, the search for the optimal solution is carried out in two steps. Firstly, a set of nondominated solutions is obtained by exploring the optimal Pareto frontier using different $\alpha$ cut level. To study the influence of fuzzy parameters on the obtained Pareto optimal solutions, all the range of the parameter fluctuation were scanned, two bounds of Alpha value have been considered $\alpha = 0, \alpha = 1$, and also we take some values between these bounds $\alpha = 0.2, \alpha = 0.4, \alpha = 0.6, \alpha = 0.8$. MM-ACO is employed to deal with this network problem. Graphical presentations of the experimental results on the six instances problem and are presented figure 6.

Fig. 6. Pareto optimal set for different level
6. EVALUATION METHOD

In order to establish the viability of our algorithm, we first compare the results obtained by running our algorithms with the results obtained from applying particle swarm optimization [4,5]. Table 3 summarizes the quality of solutions obtained by the two algorithms for the six instances. To test the effectiveness of our procedure we give in the table below the experiments results for our proposed approach for the land reclamation instances. In this table we present the results of the proposed algorithm and the results from applying PSO. For each instance we provide the best, average values. From this table, we conclude that the results obtained by the proposed algorithm are the best compared with results obtained with PSO. Table 4, reports the total and average value for each algorithm, we conclude that the values produced by the proposed algorithm are the best compared to the results produced PSO. Assuming these results is normally distributed due to large number of solutions which generated randomly.

To confirm this analysis we perform a statistical test to analyze if there is a statistically significant difference between the performances of the two algorithms. We wish to test the hypothesis where $\mu_1$ and $\mu_2$ are the average cost function obtained by ACO method and the PSO algorithm respectively. Also we wish to test the hypothesis where $\mu'_1$ and $\mu'_2$ are the average profit function obtained by ACO method and the PSO algorithm respectively. For each of the 6 instances given ($\alpha=0.0, \alpha=0.2, \alpha=0.4, \alpha=0.6, \alpha=0.8, \alpha=1.0$) we compare the two methods using the Normal Z-test, we consider the variable

$$Z = \frac{\bar{d} - d}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

With normal distribution $N(0,1)$, where $\bar{d} = \bar{x} - \bar{y}$ is the average difference between the variables of the two samples, $d = \mu_1 - \mu_2$ is the expected value, and $\sigma_1^2 + \sigma_2^2$ is the standard deviation of $\bar{d}$. Thus it appears from this table that in all cases we accept $H_0$ with Level of significance $\beta = 0.05, \beta = 0.01$.

For the cost function $H_0$ is rejected if the observed value is larger than $Z_{1-\beta}$ ($Z_{0.05} = 1.645, Z_{0.01} = 2.33$).

For the cost function, the calculated average solution is $Z = -0.46 < Z_{0.05} = 1.645$ for $\alpha = 0.05$ and for $\alpha = 0.01$ the calculated average solution is $Z = -0.46 < Z_{0.09} = 2.33$.

Therefore we accept $H_0$ on the other hand, the calculated best solution is $Z = -1.2 < Z_{0.05} = 1.645$ for $\alpha = 0.05$ and for $\alpha = 0.01$ the calculated best solution is $Z = -1.2 < Z_{0.09} = 2.33$, as similar done with the profit function knowing that $H'_{01}$ is rejected if the observed value is less than $-Z_{1-\beta}$ ($Z_{0.05} = -1.645, Z_{0.09} = -2.33$). Therefore we accept $H_{01}$ Thus these tests suggest that the proposed algorithm is significantly better than PSO algorithm.
Table 3: Computational results according to the instances for different alpha level

<table>
<thead>
<tr>
<th>instances</th>
<th>Ant colony optimization</th>
<th></th>
<th></th>
<th>Particle swarm optimization</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1(\alpha)$</td>
<td>$f_2(\alpha)$</td>
<td>$f_1(\alpha)$</td>
<td>$f_2(\alpha)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.0$</td>
<td>2001</td>
<td>433</td>
<td>1083</td>
<td>572</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.2$</td>
<td>2126</td>
<td>441</td>
<td>1076</td>
<td>570</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.4$</td>
<td>2119</td>
<td>428</td>
<td>1094</td>
<td>566</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.6$</td>
<td>2162</td>
<td>429</td>
<td>1100</td>
<td>564</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 0.8$</td>
<td>2194</td>
<td>440</td>
<td>1101</td>
<td>560</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha = 1.0$</td>
<td>2157</td>
<td>435</td>
<td>1110</td>
<td>556</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Normal Z test between ACO and PSO

<table>
<thead>
<tr>
<th>Approach</th>
<th>Average solution</th>
<th>Best solution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1(\alpha)$</td>
<td>$f_2(\alpha)$</td>
<td>$f_1(\alpha)$</td>
</tr>
<tr>
<td>ACO</td>
<td>2126.5</td>
<td>434.3333</td>
<td>67.1</td>
</tr>
<tr>
<td>PSO</td>
<td>2147.833</td>
<td>438.6667</td>
<td>89.6</td>
</tr>
</tbody>
</table>

Finally, the feasibility of using the proposed test to handle multiobjective evolutionary algorithms has been empirically approved.

7. CONCLUSIONS

Ant colony optimization has been and continues to be a fruitful paradigm for designing effective combinatorial optimization solution algorithms. The experimental analysis on the performance of a proposed method is a crucial and necessary task to carry out in this research. This paper is focused on the statistical analysis of the results in the field of population-based-techniques. Our approach has two characteristic features. Firstly, a set of nondominated solutions is obtained by exploring the optimal Pareto frontier using different $\alpha$ cut level and subsequently, based on the parametric statistical test, performance measure for the quality assessment was implemented in order to establish the viability of our algorithm, and to compare the results obtained by running our algorithms with the results obtained from applying particle swarm optimization. Furthermore, we provided an example of optimum utilization of human resources in reclamation of derelict land. The performance improvement of evolution of evolutionary algorithms still remain in the experimental stage for lack of solid theoretical support; thus, for future work, we intend to improve other statistical test.

REFERENCES


