ON g*α-I-CONTINUOUS IN TOPOLOGICAL SPACES

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Abstract: In this paper we introduce and study the notions of g*α-I-closed sets and g*α-I-continuity in Ideal topological spaces.

Keywords: g*α-I-closed, g*α-I-open and g*α-I-continuous, g*α-I-irresolute.

1. Introduction and Preliminaries

An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties. (1) A ∈ I and B ⊆ A implies B ∈ I, (2) A ∈ I and B ∈ I implies A ∪ B ∈ I. An ideal topological space is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I). For a subset A ⊆ X, A*(I, τ)={x ∈ X : A ∩ U ∉ I for every U ∈ τ (X,x)} is called the local function of A with respect to I and τ [4]. We simply write A* in case there is no chance for confusion.

A Kuratowski closure operator cl*(.) for a topology τ*(I, τ) called the *-topology, finer than τ is defined by cl*(A) = A ∪ A* [12]. If A ⊆ X, cl(A) and int(A) will respectively, denote the closure and interior of A in (X, τ).

Definition 1.1. A subset A of a topological space (x, τ) is
1. α-closed [8] if cl (int(cl(A))) ⊆ A.
2. regular closed [10], if A = cl(int(A))
3. αg closed [7], if αcl(A) ⊆ U whenever A ⊆ U and U is open in (x, τ).
4. gα-closed [7], if αcl(A) ⊆ U whenever A ⊆ U and U is α-open in (x, τ).
5. wgα-closed [11], if αcl(int(A)) ⊆ U whenever A ⊆ U and U is α-open in (x, τ).
6. wgα-closed [11], if αcl(int(A)) ⊆ U whenever A ⊆ U and U is open in (x, τ).
7. Semi closed [6], if int(cl(A)) ⊆ A.
8. g-closed [6], if cl(A) ⊆ U whenever A ⊆ U and U is open.
9. gs-closed [1], if αcl(A) ⊆ U whenever A ⊆ U and U is open.
10. αg – closed [14], if αcl(A) ⊆ U whenever A ⊆ U and U is regular open.
11. gpr-closed [3], if pcl(A) ⊆ U whenever A ⊆ U and U is regular open.
12. g#-closed [13], if cl(A) ⊆ U whenever A ⊆ U and U is αg-open.
13. g* α closed [15], if acl(A) ⊆ G whenever A ⊆ G and G is ga-open in (x, τ).

**Definition 1.2.** A function \( f : (X, \tau) \to (Y, \sigma) \) is said to be

1. \( \alpha \)-continuous [9], if for each open set \( v \in \sigma \), \( f^{-1}(v) \) is \( \alpha \)-open in \( (X, \tau) \).
2. \( \alpha \) g-continuous [2], if for each open set \( v \in \sigma \), \( f^{-1}(v) \) is \( \alpha \) g-open in \( (X, \tau) \).
3. \( g \alpha \) -continuous [2], if for each open set \( v \in \sigma \), \( f^{-1}(v) \) is \( g \alpha \) -open in \( (X, \tau) \).
4. \( w g \alpha \) -continuous [16], if for each open set \( v \in \sigma \), \( f^{-1}(v) \) is \( w g \alpha \) -open in \( (X, \tau) \).
5. \( w \alpha \) g-continuous [16], if for each open set \( v \in \sigma \), \( f^{-1}(v) \) is \( w \alpha \) g-open in \( (X, \tau) \).

2. \( g^* \alpha \)-I-closed set

**Definition 2.1** A subset \( A \) of an Ideal Topological space \((x, \tau, I)\) is said to be, \( g^* \alpha \)-I-closed set \( \alpha cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is ga-open in \( x \).

The complement of \( g^* \alpha \)-I-closed set is \( g^* \alpha \)-I-open.

**Proposition 2.2** Every closed set is \( g^* \alpha \)-I-closed set but not conversely.

Proof. Let \( A \) be a closed set and let \( U \) be ga-open set containing \( A \). Then \( A \subseteq U \). This implies that \( cl(A) \subseteq U \). Also \( \alpha cl(A) \subseteq cl(A) \subseteq U \). Therefore \( \alpha cl(A) \subseteq U \). Hence \( A \) is \( g^* \alpha \)-I-closed.

**Example 2.3** Let \( x = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{a, b\}, x\} \) and \( I = \{\phi, \{c\}\} \). Then \( A = \{b\} \) is \( g^* \alpha \)-I-closed but not in closed set.

**Proposition 2.4** Every regular closed set in \( g^* \alpha \)-I-closed set but not conversely.

Proof. Let \( A \) be regular closed set and let \( U \) be ga-open set containing \( A \). Then \( A \subseteq U \). This implies that \( cl(int(A)) \subseteq U \). \( cl(int(A)) \subseteq cl(int(cl(A))) \subseteq U \) and \( AUcl(int*(cl(A))) \subseteq cl(int(cl(A))) \).

But \( AUcl (int*(cl (A))) = \alpha cl(A) \subseteq U \). Therefore \( A \) is \( g^* \alpha \)-I-closed.

**Example 2.5** Let \( x = \{a, b, c\} \), \( \tau = \{\phi, \{b\}, \{c\}, \{b, c\}, x\} \) and \( I = \{\phi, \{c\}\} \). Then \( A = \{a\} \) is \( g^* \alpha \)-I-closed but not in regular closed.

**Proposition 2.6** Every \( \alpha \)-closed set is \( g^* \alpha \)-I-closed set but not conversely.

Proof. Assume that \( A \) is \( \alpha \)-closed set and let \( U \) be ga-open set containing \( A \). Then \( A \subseteq U \). This implies that \( acl(A) \subseteq U \). Since every \( \alpha I \)-closed set is \( \alpha \)-closed. Then \( \alpha acl(A) \subseteq acl(A) \subseteq U \). Therefore \( \alpha acl(A) \subseteq U \). Hence \( A \) is \( g^* \alpha \)-I-closed.

**Example 2.7** Let \( x = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{a, b\}, x\} \) and \( I = \{\phi, \{c\}\} \). Then \( A = \{a, c\} \) is \( g^* \alpha \)-I-closed but not in \( \alpha \)-closed.

**Proposition 2.8** Every \( g^* \alpha \)-I-closed set is \( \alpha \)-closed, \( \alpha g \)-closed set but not conversely.

**Example 2.9** Let \( x = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, x\} \) and \( I = \{\phi, \{a\}\} \). Then \( A = \{b\} \) is \( \alpha \)-closed and \( \alpha g \)-closed but not in \( g^* \alpha \)-I-closed.

**Proposition 2.10** Every \( g^* \alpha \)-I-closed is \( wg \alpha \)-closed and \( w \alpha g \)-closed set but not conversely.

**Example 2.11** Let \( x = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, x\} \) and \( I = \{\phi, \{c\}\} \). Then \( A = \{c\} \) is \( wg \alpha \)-closed, \( w \alpha g \)-closed but not in \( g^* \alpha \)-I-closed.

**Remark 2.12** The following examples shows that the concept of semi-closed and \( g^* \alpha \)-I-closed sets are independent.

**Example 2.13** Let \( x = \{a, b, c\} \), \( \tau = \{\phi, \{b\}, \{c\}, \{b, c\}, x\} \) and \( I = \{\phi, \{a\}\} \). Then \( A = \{b\} \) is semi closed but not \( g^* \alpha \)-I-closed set.

**Example 2.14** Let \( x = \{a, b, c\} \), \( \tau = \{\phi, \{a\}, \{a, b\}, x\} \) and \( I = \{\phi, \{a\}\} \) Then \( A = \{a, c\} \) is \( g^* \alpha \)-I-closed set but not semi closed.
Remark 2.15 The following example shows that the concept of g-closed and $g^\alpha$-I-closed sets are independent.

Example 2.16 Let $x = \{a, b, c\}$, $\tau = \{\{a\}, x\}$ and $I = \{\{a\}\}$. Then $A = \{a, b\}$ is g-closed but not $g^\alpha$-I-closed.

Example 2.17 Let $x = \{a, b, c\}$, $\tau = \{\{a\}, \{a, b\}, x\}$ and $I = \{\{a\}\}$. Then $A = \{b\}$ is $g^\alpha$-I-closed but not g-closed.

Proposition 2.18 Every g-closed set is $g^\alpha$-I-closed but not conversely.

Proof. Let A be g-closed set and U be a regular open set containing A. Then U is $g^\alpha$-open set containing A. Hence U and B are both g-open set. Therefore A is $g^\alpha$-I-closed.

Example 2.19 Let $x = \{a, b, c\}$, $\tau = \{\{a\}, \{a, b\}\}$ and $I = \{\{a\}\}$. Then $A = \{a\}$ is g-closed but not in $g^\nu$-closed.

Proposition 2.20 Every $g^\alpha$-I-closed set is $gs$-closed but not conversely.

Proof. Let a subset A be $g^\alpha$-I-closed and U be an open set containing A. Then U is $g^\nu$-open set containing A. Since A is $g^\nu$-closed. Therefore cl(A) $\subseteq$ U. Hence every closed set is $g^\alpha$-closed set.

Example 2.21 Let $x = \{a, b, c\}$, $\tau = \{\{a\}, \{a, b\}, \{a, b\}, x\}$ and $I = \{\{a\}\}$. Then $A = \{a\}$ is g-closed but not in $g^\alpha$-I-closed.

Proposition 2.22 Every $g^\alpha$-I-closed set is $gr$-closed but not conversely.

Proof. Assume that A is $g^\alpha$-I-closed set in $(x, \tau, I)$ and let U be regular open set. Since every regular open set is $g^\nu$-open set. Therefore A $\subseteq$ U. Hence A is $gr$-closed.

Example 2.23 Let $x = \{a, b, c\}$, $\tau = \{\{a\}, \{a, b\}\}$ and $I = \{\{a\}\}$. Then $A = \{a\}$ is $gr$-closed but not in $g^\alpha$-I-closed.

Proposition 2.24 Every $g^\alpha$-I-closed set is gpr-closed.

Proof. By above proposition $g^\alpha$-I-closed set is $g^\nu$-closed. But every $g^\nu$-closed set is gpr-closed. Hence every $g^\alpha$-I-closed set is gpr-closed.

Proposition 2.25 Every $g^\alpha$-I-closed set is $g^\alpha$-closed set.

Proof. Let A be $g^\alpha$-I-closed in $(x, \tau, I)$. Then we have

$\alpha cl(A)$ whenever $A \subseteq U$ and U is $g^\nu$-open

- $A \subseteq \{\text{int}(\text{cl}(\text{int}(A)))\} \subseteq \text{int}(\text{cl}(\text{int}(A)))$
- $\alpha cl(A)$

This shows that A is $g^\alpha$-closed.

Example 2.26 Let $x = \{a, b, c\}$, $\tau = \{\{a\}, x\}$ and $I = \{\{c\}\}$. Then $A = \{b\}$ is $g^\alpha$-closed but not $g^\alpha$-I-closed.

Proposition 2.27 Union of two $g^\alpha$-I-closed sets is $g^\alpha$-I-closed.

Proof. Let A and B be $g^\alpha$-I-closed in X. Let U be a $g^\nu$-open in X. Such that $U$. Then $A \subseteq U$ and $B \subseteq U$. Hence $\alpha cl(A \cup B) = \alpha cl(A) \cup \alpha cl(B) \subseteq U$. Therefore $A \cup B$ is $g^\alpha$-I-closed.

Remark 2.28 i) The intersection of any two $g^\alpha$-I-closed set is $g^\alpha$-I-closed set.

ii) Suppose $I = \{\phi\}$, then the notion of $g^\alpha$-I-closed set coincide with $g^\alpha$-closed set.

Remark 2.29 For the subsets defined above, we have the following implications.
None of the implications is reversible.

3. **g**\(^*\)α-I-continuity

**Definition 3.1.** A function \(f:(X,\tau,I)\rightarrow(Y,\sigma)\) is said to be \(g\)\(^*\)α-I-continuous if the inverse image of every closed set in \((Y,\sigma)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\).

**Definition 3.2.** A function \(f:(X,\tau,I)\rightarrow(Y,\sigma,J)\) is said to be \(g\)\(^*\)α-I-irresolute, if the inverse image of every \(g\)\(^*\)α-I-closed set in \((Y,\sigma,J)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\).

**Remark 3.3.** If \(I=\{\emptyset\}\), the notion of \(g\)\(^*\)α-I-continuous coincides with the notion of \(g\)\(^*\)α-continuous.

**Theorem 3.4.**

i) Every continuous function is \(g\)\(^*\)α-I-continuous.

ii) Every α-continuous function is \(g\)\(^*\)α-I-continuous.

Proof: i) Assume that \(f:(X,\tau,I)\rightarrow(Y,\sigma)\) is a continuous function. Let \(V\) be any closed set in \((Y,\sigma)\). Then \(f^{-1}(V)\) is closed. Since every closed is \(g\)\(^*\)α-I-closed set. Hence \(f^{-1}(x)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\). Therefore \(f\) is \(g\)\(^*\)α-I-continuous.

ii) Since every continuous function is α-continuous. Therefore \(f\) is \(g\)\(^*\)α-I-continuous.

**Remark 3.5.** The above theorem need not be true as seen from the following examples.

**Example 3.6.** Let \(X=\{a,b,c\}, \tau=\{\phi,a,\{a,b\},X\}, \sigma=\{\phi,\{b\},X\}\) and \(I=\{\phi,\{c\}\}\). Let \(f:(X,\tau,I)\rightarrow(X,\sigma)\) be defined by \(f(a)=a, f(b)=b, f(c)=c\) then \(f\) is \(g\)\(^*\)α-I-continuous but not continuous.

**Example 3.7.** Let \(X=\{a,b,c\}, \tau=\{\phi,\{a\},\{a,b\},X\}, \sigma=\{\phi,\{a,c\},X\}\) and \(I=\{\phi,\{c\}\}\). Let \(f:(X,\tau,I)\rightarrow(X,\sigma)\) be defined by \(f(a)=a, f(b)=b, f(c)=c\) then \(f\) is \(g\)\(^*\)α-I-continuous but not α-continuous.

**Theorem 3.8.** A map \(f:(X,\tau,I)\rightarrow(Y,\sigma)\) is \(g\)\(^*\)α-I-continuous iff the inverse image of every closed set is \((Y,\sigma)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\).

Proof: **Necessity:** Let \(V\) be an open set in \((Y,\sigma)\). Since \(f\) is \(g\)\(^*\)α-I-continuous, \(f^{-1}(V)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\). But \(f^{-1}(V)=X=f^{-1}(V)\). Hence \(f^{-1}(V)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\).

**Sufficiency:** Assume that the inverse image of every closed set in \((Y,\sigma)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\). Let \(V\) be a closed set in \((Y,\sigma)\). By our assumption \(f^{-1}(V)=X=f^{-1}(V)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\) which implies that \(f^{-1}(V)\) is \(g\)\(^*\)α-I-closed in \((X,\tau,I)\). Hence \(f\) is \(g\)\(^*\)α-I-continuous.

**Theorem 3.9.** For a function \(f:(X,\tau,I)\rightarrow(Y,\sigma)\), the following hold

1. Every \(g\)\(^*\)α-I-continuous function is $\alpha$-continuous.
2. Every \(g\)\(^*\)α-I-continuous function is $\alpha g$-continuous.
3. Every \(g\)\(^*\)α-I-continuous function is $\omega g\alpha$-continuous.
4. Every \(g\)\(^*\)α-I-continuous function is $\omega g\alpha$-continuous.
Remark 3.10. The converses of the above theorem need not be true as seen from the following examples.

Example 3.11. Let \( X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}, \sigma = \{\phi, \{a, b\}, X\} \) and \( I = \{\phi, \{a\}\} \). Let the function \( f: (X, \tau, I) \rightarrow (X, \sigma) \) be defined by \( f(a) = a, f(b) = b, f(c) = c \) then the function \( f \) is \( g \alpha \)-continuous, \( w g \alpha \)-continuous, \( w g \alpha \)-continuous but not \( g^* \alpha \)-I-continuous.

Example 3.12. Let \( X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{a, b\}, X\}, \sigma = \{\phi, \{a\}, X\} \) and \( I = \{\phi, \{a\}\} \). Let the function \( f: (X, \tau, I) \rightarrow (X, \sigma) \) be defined by \( f(a) = a, f(b) = b, f(c) = c \) then the function \( f \) is \( g \alpha \)-continuous but not \( g^* \alpha \)-I-continuous.

Theorem 3.13. For a function \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) the following hold.
1. Every \( g^* \alpha \)-I-continuous function is \( g \alpha \)-I-continuous.
2. Every \( g^* \alpha \)-I-continuous function is \( a g \alpha \)-I-continuous.
3. Every \( g \alpha \)-I-continuous function is \( w g \alpha \)-I-continuous.
4. Every \( g^* \alpha \)-I-continuous function is \( w g \)-I-continuous.

Remark 3.14. The converses of the above examples need not be true as seen from the following examples.

Example 3.15. i) Let \( X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{a, b\}, X\}, I = \{\phi, \{a\}\} \) and \( \sigma = \{\phi, \{a\}, X\} \). Let the function \( f: (X, \tau, I) \rightarrow (X, \sigma) \) be defined by \( f(a) = a, f(b) = b, f(c) = c \) then the function \( f \) is \( a g \alpha \)-continuous, \( w g \alpha \)-I-continuous, \( g \alpha \)-I-continuous but not \( g^* \alpha \)-I-continuous.

ii) Let \( X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{a, c\}, X\}, I = \{\phi, \{a\}\} \) and \( \sigma = \{\phi, \{c\}, X\} \). Let the function \( f: (X, \tau, I) \rightarrow (X, \sigma) \) be defined by \( f(a) = a, f(b) = b, f(c) = c \) then the function \( f \) is \( g \alpha \)-continuous but not \( g^* \alpha \)-I-continuous.

Theorem 3.16. Let \( f: X \rightarrow Y \) be a map. Then the following statements are equivalent:
(i) \( f \) is \( g^* \alpha \)-I-continuous.
(ii) the inverse image of each open set in \( Y \) is \( g^* \alpha \)-I-open in \( X \).

Proof: Assume that \( f: X \rightarrow Y \) is \( g^* \alpha \)-I-continuous. Let \( G \) be open in \( Y \). The \( G^c \) is closed in \( Y \). Since \( f \) is \( g^* \alpha \)-I-continuous, \( f^{-1}(G^c) \) is \( g^* \alpha \)-I-closed in \( X \). But \( f^{-1}(G^c) = X - f^{-1}(G) \). Thus \( f^{-1}(G) = g^* \alpha \)-I-open in \( X \).

Conversely assume that the inverse image of each open set in \( Y \) is \( g^* \alpha \)-I-open in \( X \). Let \( F \) be any closed set in \( Y \). By assumption \( c \) is \( g^* \alpha \)-I-open in \( X \). But \( f^{-1}(F^c) = X - f^{-1}(F) \). Thus \( X - f^{-1}(F) = g^* \alpha \)-I-open in \( X \) and so \( f^{-1}(F) \) is \( g^* \alpha \)-I-closed in \( X \). Therefore \( f \) is \( g^* \alpha \)-I-continuous. Hence (i) and (ii) are equivalent.

Theorem 3.17. Let \( X = A \cup B \) be a topological space with topology \( \tau \) and \( Y \) be a topological space with topology \( \sigma \). Let \( f: (A, \tau/A) \rightarrow (Y, \sigma) \) and \( g: (B, \tau/B) \rightarrow (Y, \sigma) \) be \( g^* \alpha \)-I-continuous maps such that \( f(x) = g(x) \) for every \( x \in A \cap B \). Suppose that \( A \) and \( B \) are \( g^* \alpha \)-closed sets in \( X \). Then the combination \( \alpha: (X, \tau, I) \rightarrow (Y, \sigma) \) is \( g^* \alpha \)-I-continuous.

Proof: Let \( F \) be any closed set in \( Y \). Clearly \( \alpha^{-1}(F) = f^{-1}(F) \cup g^{-1}(F) = C \cup D \) where \( C = f^{-1}(F) \) and \( D = g^{-1}(F) \).

But \( C \) is \( g^* \alpha \)-I-closed in \( A \) and \( A \) is \( g^* \alpha \)-I-closed in \( Y \) and so \( C \) is \( g^* \alpha \)-closed in \( X \). Since we have proved that if \( B \subseteq A \subseteq X \), \( B \) is \( g^* \alpha \)-I-closed in \( A \) and \( A \) is \( g^* \alpha \)-I-closed in \( X \). Also \( C \cup D \) is \( g^* \alpha \)-I-closed in \( X \). Therefore \( \alpha^{-1}(F) \) is \( g^* \alpha \)-I-closed in \( X \). Hence \( \alpha \) is \( g^* \alpha \)-I-continuous.

Theorem 3.18. Let \( f: (X, \tau, I) \rightarrow (Y, \sigma) \) is \( g^* \alpha \)-I-continuous and \( g: (Y, \sigma) \rightarrow (Z, \eta) \) is continuous, then \( g \circ f: (X, \tau, I) \rightarrow (Z, \eta) \) is \( g^* \alpha \)-I-continuous.

Proof. Let \( g \) be a continuous function and \( v \) be any open set in \( (z, \eta) \) then \( f^{-1}(v) \) is open in \( (z, \eta) \) then \( f^{-1}(v) \) is open in \( (Y, \sigma) \). Since \( f \) is \( g^* \alpha \)-I-continuous. \( f^{-1}(g^{-1}(v)) = (g \circ f)^{-1}(v) \) is \( g^* \alpha \)-I-open in \( (X, \tau, I) \). Hence \( (g \circ f) \) is \( g^* \alpha \)-I-continuous.

Theorem 3.19. Let \( f: (X, \tau, I) \rightarrow (Y, \sigma, J) \) and \( g: (Y, \sigma, J) \rightarrow (Z, \eta, K) \) are \( g^* \alpha \)-I-irresolute then \( g \circ f: (X, \tau, I) \rightarrow (Z, \eta, K) \) is \( g^* \alpha \)-I-irresolute.
Proof. Let $g$ be a $g^\ast\alpha$-$\alpha$-I-irresolute function and $v$ be any $g^\ast\alpha - K$-open in $(Z, \eta, K)$, then $f^{-1}(V)$ is $g^\ast\alpha - J$-open in $(Y, \sigma, J)$.

Since $f$ is $g^\ast\alpha$-$\alpha$-I-irresolute, $f^{-1}(g^{-1}(v)) = (g \circ f)^{-1}(v)$ is $g^\ast\alpha - I$-open in $(X, \tau, I)$. Hence $(g \circ f)$ is $g^\ast\alpha$-$\alpha$-I-irresolute.

**Theorem 3.20.** Let $f: X \to Y$ and $g: Y \to Z$ be any two functions. Let $h = g \circ f$. Then:

(i) $h$ is $g^\ast\alpha$-$\alpha$-I-continuous if $f$ is $g^\ast\alpha$-$\alpha$-I-irresolute and $g$ is $g^\ast\alpha$-$\alpha$-I-continuous,

(ii) $h$ is $g^\ast\alpha$-$\alpha$-I-continuous if $g$ is continuous and $f$ is $g^\ast\alpha$-$\alpha$-I-continuous.

**Proof.** Let $V$ be closed in $Z$. (i) Suppose $f$ is $g^\ast\alpha$-$\alpha$-I-irresolute and $g$ is $g^\ast\alpha$-$\alpha$-I-continuous. Since $g$ is $g^\ast\alpha$-$\alpha$-I-continuous, $g^{-1}(V)$ is $g^\ast\alpha$-$\alpha$-I-closed in $Y$. Since $f$ is $g^\ast\alpha$-$\alpha$-I-irresolute, by definition, $f^{-1}(g^{-1}(V))$ is $g^\ast\alpha$-$\alpha$-I-closed in $X$. This proves (i).

(ii) Let $g$ be continuous and $f$ be $g^\ast\alpha$-$\alpha$-I-continuous. Then $g^{-1}(V)$ is closed in $Y$. Since $f$ is $g^\ast\alpha$-$\alpha$-I-continuous, using the definition, $f^{-1}(g^{-1}(V))$ is $g^\ast\alpha$-$\alpha$-I-closed in $X$. This proves (iii).

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